Classification Algorithms – Continued

Outline

- **Rules**
- Linear Models (Regression)
- Instance-based (Nearest-neighbor)

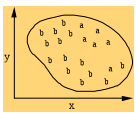
Generating Rules

- Decision tree can be converted into a rule set
- Straightforward conversion:
  - each path to the leaf becomes a rule – makes an overly complex rule set
- More effective conversions are not trivial
  - (e.g. C4.8 tests each node in root-leaf path to see if it can be eliminated without loss in accuracy)

Covering algorithms

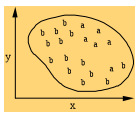
- Strategy for generating a rule set directly: for each class in turn find rule set that covers all instances in it (excluding instances not in the class)
- This approach is called a *covering* approach because at each stage a rule is identified that covers some of the instances

Example: generating a rule

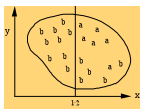


If true then class = a

Example: generating a rule, II

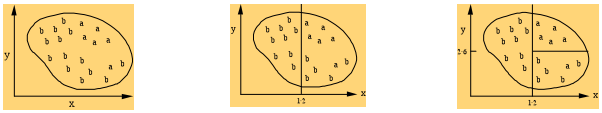


If true then class = a



If x > 1.2 then class = a

Example: generating a rule, III

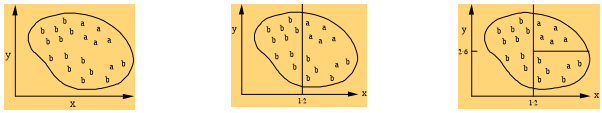


If true then class = a

If x > 1.2 and y > 2.6 then class = a

If x > 1.2 then class = a

Example: generating a rule, IV



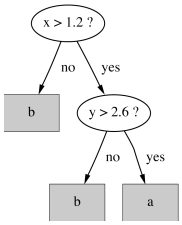
If true then class = a

If x > 1.2 and y > 2.6 then class = a

If x > 1.2 then class = a

- Possible rule set for class "b":
  If x ≤ 1.2 then class = b
  If x > 1.2 and y ≤ 2.6 then class = b
- More rules could be added for "perfect" rule set

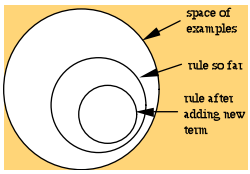
Rules vs. trees



- Corresponding decision tree:
  (produces exactly the same predictions)
- But: rule sets *can* be more clear when decision trees suffer from replicated subtrees
- Also: in multi-class situations, covering algorithm concentrates on one class at a time whereas decision tree learner takes all classes into account

A simple covering algorithm

- Generates a rule by adding tests that maximize rule's accuracy
- Similar to situation in decision trees: problem of selecting an attribute to split on
  - But: decision tree inducer maximizes overall purity
- Each new test reduces rule's coverage:



Selecting a test

- Goal: maximize accuracy
  - *t* total number of instances covered by rule
  - *p* positive examples of the class covered by rule
  - *t* – *p* number of errors made by rule
  - ⇒ Select test that maximizes the ratio *p*/*t*
- We are finished when *p*/*t* = 1 or the set of instances can't be split any further

Example: contact lens data

- Rule we seek:

```
If ?
    then recommendation = hard
```

- Possible tests:

Age = Young	2/8
Age = Pre-presbyopic	1/8
Age = Presbyopic	1/8
Spectacle prescription = Myope	3/12
Spectacle prescription = Hypermetrope	1/12
Astigmatism = no	0/12
Astigmatism = yes	4/12
Tear production rate = Reduced	0/12
Tear production rate = Normal	4/12

Modified rule and resulting data

- Rule with best test added:

```
If astigmatism = yes
    then recommendation = hard
```

- Instances covered by modified rule:

Age	Spectacle prescription	Astigmatism	Tear production rate	Recommended lenses
Young	Myope	Yes	Reduced	None
Young	Myope	Yes	Normal	Hard
Young	Hypermetrope	Yes	Reduced	None
Young	Hypermetrope	Yes	Normal	hard
Pre-presbyopic	Myope	Yes	Reduced	None
Pre-presbyopic	Myope	Yes	Normal	Hard
Pre-presbyopic	Hypermetrope	Yes	Reduced	None
Pre-presbyopic	Hypermetrope	Yes	Normal	None
Presbyopic	Myope	Yes	Reduced	None
Presbyopic	Myope	Yes	Normal	Hard
Presbyopic	Hypermetrope	Yes	Reduced	None
Presbyopic	Hypermetrope	Yes	Normal	None

Further refinement

- Current state:

```
If astigmatism = yes
   and ?
  then recommendation = hard
```

- Possible tests:

```
Age = Young                          2/4
Age = Pre-presbyopic                 1/4
Age = Presbyopic                     1/4
Spectacle prescription = Myope       3/6
Spectacle prescription = Hypermetrope 1/6
Tear production rate = Reduced       0/6
Tear production rate = Normal        4/6
```


Modified rule and resulting data

- Rule with best test added:

```
If astigmatism = yes
    and tear production rate = normal
  then recommendation = hard
```

- Instances covered by modified rule:

Age	Spectacle prescription	Astigmatism	Tear production rate	Recommended lenses
Young	Myope	Yes	Normal	Hard
Young	Hypermetrope	Yes	Normal	hard
Pre-presbyopic	Myope	Yes	Normal	Hard
Pre-presbyopic	Hypermetrope	Yes	Normal	None
Presbyopic	Myope	Yes	Normal	Hard
Presbyopic	Hypermetrope	Yes	Normal	None

Further refinement

- Current state:

```
If astigmatism = yes
    and tear production rate = normal
    and ?
  then recommendation = hard
```

- Possible tests:

```
Age = Young                          2/2
Age = Pre-presbyopic                 1/2
Age = Presbyopic                     1/2
Spectacle prescription = Myope       3/3
Spectacle prescription = Hypermetrope 1/3
```

- Tie between the first and the fourth test
  - We choose the one with greater coverage

The result

- Final rule:

```
If astigmatism = yes
   and tear production rate = normal
   and spectacle prescription = myope
   then recommendation = hard
```

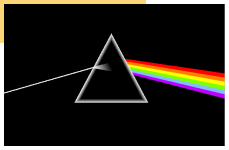
- Second rule for recommending "hard lenses":
  (built from instances not covered by first rule)

```
If age = young and astigmatism = yes
   and tear production rate = normal
   then recommendation = hard
```

- These two rules cover all "hard lenses":
  - Process is repeated with other two classes

Pseudo-code for PRISM

```
For each class C
  Initialize E to the instance set
  While E contains instances in class C
    Create a rule R with an empty left-hand side that predicts class C
    Until R is perfect (or there are no more attributes to use) do
      For each attribute A not mentioned in R, and each value v,
        Consider adding the condition A = v to the left-hand side of R
        Select A and v to maximize the accuracy p/t
          (break ties by choosing the condition with the largest p)
      Add A = v to R
    Remove the instances covered by R from E
```



Rules vs. decision lists

- PRISM with outer loop removed generates a decision list for one class
  - Subsequent rules are designed for rules that are not covered by previous rules
  - But: order doesn't matter because all rules predict the same class
- Outer loop considers all classes separately
  - No order dependence implied
- Problems: overlapping rules, default rule required

Separate and conquer

- Methods like PRISM (for dealing with one class) are *separate-and-conquer* algorithms:
  - First, a rule is identified
  - Then, all instances covered by the rule are separated out
  - Finally, the remaining instances are "conquered"
- Difference to divide-and-conquer methods:
  - Subset covered by rule doesn't need to be explored any further

Outline

- Rules
- **Linear Models (Regression)**
- Instance-based (Nearest-neighbor)

Linear models

- Work most naturally with numeric attributes
- Standard technique for numeric prediction: linear regression
  - Outcome is linear combination of attributes

$$x = w_0 + w_1 a_1 + w_2 a_2 + \ldots + w_k a_k$$

- Weights are calculated from the training data
- Predicted value for first training instance $\mathbf{a}^{(1)}$

$$w_0 a_0^{(1)} + w_1 a_1^{(1)} + w_2 a_2^{(1)} + \ldots + w_k a_k^{(1)} = \sum_{j=0}^{k} w_j a_j^{(1)}$$

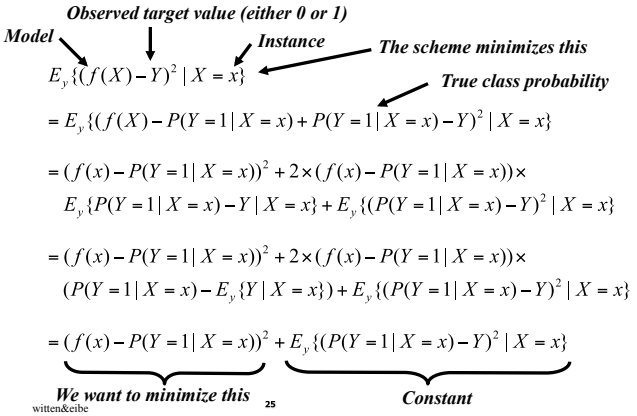
Minimizing the squared error

- Choose $k+1$ coefficients to minimize the squared error on the training data
- Squared error: $\sum_{i=1}^{n} \left(x^{(i)} - \sum_{j=0}^{k} w_j a_j^{(i)} \right)^2$
- Derive coefficients using standard matrix operations
- Can be done if there are more instances than attributes (roughly speaking)
- Minimizing the *absolute error* is more difficult

Regression for Classification

- *Any* regression technique can be used for classification
  - Training: perform a regression for each class, setting the output to 1 for training instances that belong to class, and 0 for those that don't
  - Prediction: predict class corresponding to model with largest output value (*membership value*)
- For linear regression this is known as *multi-response linear regression*

*Theoretical justification



*Pairwise regression

- Another way of using regression for classification:
  - A regression function for every *pair* of classes, using only instances from these two classes
  - Assign output of +1 to one member of the pair, –1 to the other
- Prediction is done by voting
  - Class that receives most votes is predicted
  - Alternative: "don't know" if there is no agreement
- More likely to be accurate but more expensive

Logistic regression

- Problem: some assumptions violated when linear regression is applied to classification problems
- *Logistic* regression: alternative to linear regression
  - Designed for classification problems
  - Tries to estimate class probabilities directly
    - Does this using the *maximum likelihood* method
  - Uses this linear model:

$$\log\left(\frac{P}{1-P}\right) = w_0 a_0 + w_1 a_1 + w_2 a_2 + \ldots + w_k a_k$$

***P*= Class probability**

Discussion of linear models

- Not appropriate if data exhibits non-linear dependencies
- But: can serve as building blocks for more complex schemes (i.e. model trees)
- Example: multi-response linear regression defines a *hyperplane* for any two given classes:

$$(w_0^{(1)} - w_0^{(2)})a_0 + (w_1^{(1)} - w_1^{(2)})a_1 + (w_2^{(1)} - w_2^{(2)})a_2 + \ldots + (w_k^{(1)} - w_k^{(2)})a_k > 0$$

Comments on basic methods

- Minsky and Papert (1969) showed that linear classifiers have limitations, e.g. can't learn XOR
  - But: combinations of them can (→ Neural Nets)

Outline

- Rules
- Linear Models (Regression)
- **Instance-based (Nearest-neighbor)**

Instance-based representation

- Simplest form of learning: *rote learning*
  - Training instances are searched for instance that most closely resembles new instance
  - The instances themselves represent the knowledge
  - Also called *instance-based* learning
- Similarity function defines what's "learned"
- Instance-based learning is *lazy* learning
- Methods:
  - *nearest-neighbor*
  - *k-nearest-neighbor*
  - ...

The distance function

- Simplest case: one numeric attribute
  - Distance is the difference between the two attribute values involved (or a function thereof)
- Several numeric attributes: normally, Euclidean distance is used and attributes are normalized
- Nominal attributes: distance is set to 1 if values are different, 0 if they are equal
- Are all attributes equally important?
  - Weighting the attributes might be necessary

Instance-based learning

- Distance function defines what's learned
- Most instance-based schemes use *Euclidean distance*:

$$\sqrt{(a_1^{(1)} - a_1^{(2)})^2 + (a_2^{(1)} - a_2^{(2)})^2 + \ldots + (a_k^{(1)} - a_k^{(2)})^2}$$

  $\mathbf{a}^{(1)}$ and $\mathbf{a}^{(2)}$: two instances with *k* attributes
- Taking the square root is not required when comparing distances
- Other popular metric: *city-block (Manhattan) metric*
  - Adds differences without squaring them

Normalization and other issues

- Different attributes are measured on different scales ⇒ need to be *normalized*:

$$a_i = \frac{v_i - \min v_i}{\max v_i - \min v_i} \quad \text{or} \quad a_i = \frac{v_i - Avg(v_i)}{StDev(v_i)}$$

  v_i: the actual value of attribute *i*
- Nominal attributes: distance either 0 or 1
- Common policy for missing values: assumed to be maximally distant (given normalized attributes)

Discussion of 1-NN

- Often very accurate
- ... but slow:
  - simple version scans entire training data to derive a prediction
- Assumes all attributes are equally important
  - Remedy: attribute selection or weights
- Possible remedies against noisy instances:
  - Take a majority vote over the *k* nearest neighbors
  - Removing noisy instances from dataset (difficult!)
- Statisticians have used *k*-NN since early 1950s
  - If $n \rightarrow \infty$ and $k/n \rightarrow 0$, error approaches minimum

Summary

- Simple methods frequently work well
  - robust against noise, errors
- Advanced methods, if properly used, can improve on simple methods
- No method is universally best