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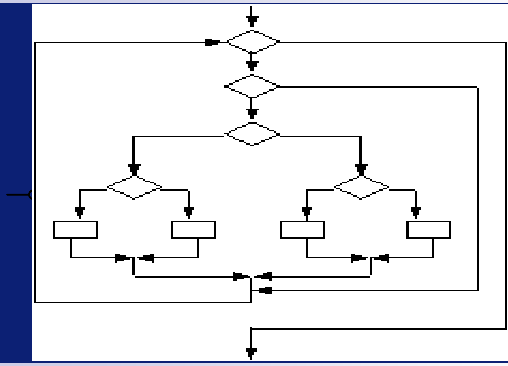

Program correctness

LTL equivalences

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LTL equivalences

- We say that two LTL formulas ϕ and ψ are semantically equivalent, writing $\phi \equiv \psi$ if for all models M and for all paths π of M we have

$$\pi \vDash \phi \text{ iff } \pi \vDash \psi$$

De Morgan-based equivalences

- $\neg F\phi \equiv G\neg\phi$
- $\neg G\phi \equiv F\neg\phi$
- $\neg X\phi \equiv X\neg\phi$ X-self duality: on a path each state has a unique successor

- $\neg(\phi \; U \; \psi) \equiv \neg\phi \; R \; \neg\psi$
- $\neg(\phi \; R \; \psi) \equiv \neg\phi \; U \; \neg\psi$

Distributivities

- $F(\phi \vee \psi) \equiv F\phi \vee F\psi$
- $G(\phi \wedge \psi) \equiv G\phi \wedge G\psi$

Reductions

- $F\phi \equiv T \; U \; \phi$
- $G\phi \equiv \bot \; R \; \phi$

- $\phi \; U \; \psi \equiv \phi \; W \; \psi \wedge F\psi$
- $\phi \; W \; \psi \equiv \phi \; U \; \psi \vee F\psi$

- $\phi \; W \; \psi \equiv \psi \; R \; (\phi \vee \psi)$
- $\phi \; R \; \psi \equiv \psi \; W \; (\phi \wedge \psi)$

LTL: Adequate sets of connectives

- A set of operators S is adequate for LTL if every formula in LTL can be expressed as an equivalent one using only the operators in S.

- Theorem: The set of operators

  $$\top, \neg, \wedge, X, U$$

  is adequate for LTL.

- Without negation, the set of operators

  $$\top, \bot, \vee, \wedge, X, U, R$$

  is adequate but $\top, \bot, \vee, \wedge, X, R, G$ is not (because one cannot define F).

Other LTL equivalences

- $G\phi \equiv \phi \wedge XG\phi$
- $F\phi \equiv \phi \vee XF\phi$
- $\phi U\psi \equiv \psi \vee (\phi \wedge X(\phi U\psi))$

- Theorem: $\phi U\psi \equiv \neg(\neg\psi U(\neg\phi \wedge \neg\psi)) \wedge F\psi$