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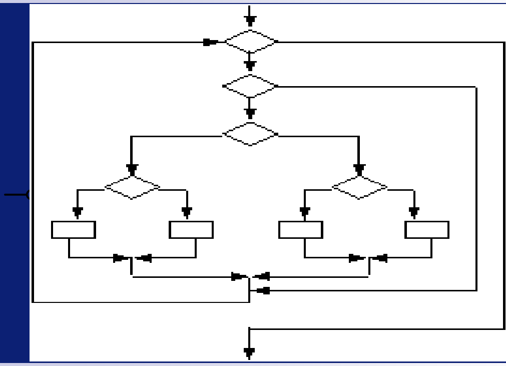

Program correctness

Using temporal logics

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LTL equivalences

- De Morgan-based
  - $\neg F\phi \equiv G\neg\phi$
  - $\neg X\phi \equiv X\neg\phi$

X-self duality: on a path each state has a unique successor

- Until reduction
  - $F\phi \equiv T\ U\ \phi$
  - $F\phi \equiv T\ U\ \phi$

LTL: Adequate sets of connectives

- Theorem: The set of operators

$$T, \neg, \wedge, U, X$$

is adequate for LTL.

  - $\phi U \psi \equiv \neg(E[\neg\psi U(\neg\phi \wedge \neg\psi)] \vee AG \neg\psi)$
  - $\phi R \psi \equiv \neg(\neg\phi U \neg\psi)$
  - $\phi W \psi \equiv \psi R(\phi \vee \psi)$

Other LTL equivalences

- $G\phi \equiv \phi \wedge XG\phi$
- $F\phi \equiv \phi \vee XF\phi$
- $\phi U\psi \equiv \psi \vee (\phi \wedge X\phi U\psi)$

- Theorem: $\phi U\psi \equiv \neg(\neg\psi U(\neg\phi \wedge \neg\psi)) \wedge F\psi$

Verification goals

- Formulating properties requires some expertise
- Today we present categories of fundamental properties commonly used for system verification
  - reachability properties
  - safety properties
  - liveness properties
  - fairness properties

Reachability

- A reachability property states that some particular situation can be reached
  - Simple
    - “We can obtain n < 0”
    - “We can enter a critical section”
  - Conditional
    - “We can enter a critical section without traversing n= 0”
  - Any
    - “we can always return to the initial state”

Reachability in LTL

- LTL misses the existential quantifier on paths, thus it can only express reachability negatively:

  something is not reachable

- Simple reachability
  - $\neg G(n \geq 0)$
  - $\neg G(no\_critic\_sec)$

Safety

- A safety property states that, under certain conditions, an event never occurs
  - “Two processes will never be both in their critical section”
  - “A memory overflow will never occur”

- In general, safety statements express that an undesirable event will not occur.
- The negation of a reachability property is a safety property (and the other way around)

Safety in LTL

- Typically expressed by the combinator G in LTL
- Examples
  - $G(\neg critic\_sec_1 \wedge \neg critic\_sec_2)$
  - $G(\neg overflow)$
- Conditional safety

  "As long the key is not in, the car won't start"
  - $\neg start \; W \; key$
  - $\neg start \; U \; key$ as we are not required to have the key in some day

Liveness

- A liveness property states that, under certain conditions, an event will ultimately occur
  - “Any request will be satisfied”
  - “The light will turn green”
  - “after the rain, the sunshine”

- Liveness is not reachability

  “The light will turn green (some day, regardless of the system behavior)”

  vs.

  “It is possible for the light (some day) to turn green”

Liveness

- In general, liveness statements express that happy event will occur in the end
- Termination is a liveness property:
  - “The program will terminate”

Liveness in LTL

- Typically expressed by the combinator F
- Examples
  - $G(req \Rightarrow Fsat)$ in LTL
- In LTL $\phi_1 U \phi_2$ is a liveness property, whereas $\phi_1 W \phi_2$ is a safety property

Deadlock

- A deadlock property states that, the system can never be in a situation in which no progress is possible

- ~~Safety?~~ Liveness?
  - Deadlock freeness in LTL

    GX T

  whatever state may be reached (G) there exists an immediate successor state (X T)

Fairness

- A fairness property states that, under certain conditions, an event will occur (or will fail to occur) infinitely often
  - "If access to a critical section is infinitely often requested, then access will be granted infinitely often

Fairness in LTL

- Typically expressed by the combinators
  - GF (infinitely often)
  - FG (eventually always)
- Examples
  - GF critic_in $\vee$ FG$\neg$ critic_req