Knowledge Representation

Outline: Output - Knowledge representation

- Decision tables
- Decision trees
- Decision rules
- Rules involving relations
- Instance-based representation
  - Prototypes, Clusters

Output: representing structural patterns

- Many different ways of representing patterns
  - Decision trees, rules, instance-based, …
- Also called "knowledge" representation
- Representation determines inference method
- Understanding the output is the key to understanding the underlying learning methods
- Different types of output for different learning problems (e.g. classification, regression, …)

Decision tables

- Simplest way of representing output:
  - Use the same format as input!
- Decision table for the weather problem:

Outlook	Humidity	Play
Sunny	High	No
Sunny	Normal	Yes
Overcast	High	Yes
Overcast	Normal	Yes
Rainy	High	No
Rainy	Normal	No

- Main problem: selecting the right attributes
- Also, not flexible enough

Decision trees

- "Divide-and-conquer" approach produces tree
- Nodes involve testing a particular attribute
- Usually, attribute value is compared to constant
- Other possibilities:
  - Comparing values of two attributes
  - Using a function of one or more attributes
- Leaves assign classification, set of classifications, or probability distribution to instances
- Unknown instance is routed down the tree

Nominal and numeric attributes

- Nominal:
  number of children usually equal to number values
  ⇒ attribute won't get tested more than once
  - Other possibility: division into two subsets
- Numeric:
  test whether value is greater or less than constant
  ⇒ attribute may get tested several times
  - Other possibility: three-way split (or multi-way split)
    - Integer: *less than, equal to, greater than*
    - Real: *below, within, above*

Missing values

- Does absence of value have some significance?
- Yes ⇒ "missing" is a separate value
- No ⇒ "missing" must be treated in a special way
  - Solution A: assign instance to most popular branch
  - Solution B: split instance into pieces
    - Pieces receive weight according to fraction of training instances that go down each branch
    - Classifications from leave nodes are combined using the weights that have percolated to them

Classification rules

- Popular alternative to decision trees
- *Antecedent* (pre-condition): a series of tests (just like the tests at the nodes of a decision tree)
- Tests are usually logically ANDed together (but may also be general logical expressions)
- *Consequent* (conclusion): classes, set of classes, or probability distribution assigned by rule
- Individual rules are often logically ORed together
  - Conflicts arise if different conclusions apply

From trees to rules

- Easy: converting a tree into a set of rules
  - One rule for each leaf:
    - Antecedent contains a condition for every node on the path from the root to the leaf
    - Consequent is class assigned by the leaf
- Produces rules that are unambiguous
  - Doesn't matter in which order they are executed
- But: resulting rules are unnecessarily complex
  - Pruning to remove redundant tests/rules

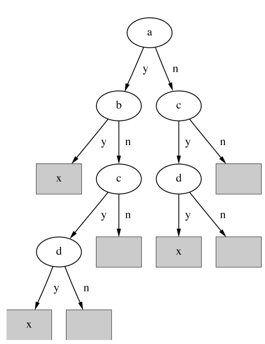
From rules to trees

- More difficult: transforming a rule set into a tree
  - Tree cannot easily express disjunction between rules
- Example: rules which test different attributes

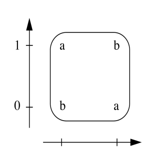
```
If a and b then x
If c and d then x
```

- Symmetry needs to be broken
- Corresponding tree contains identical subtrees (⇒ "replicated subtree problem")

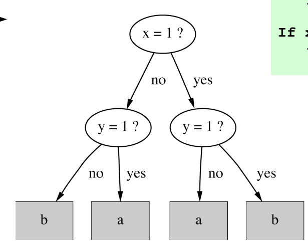
A tree for a simple disjunction



The exclusive-or problem

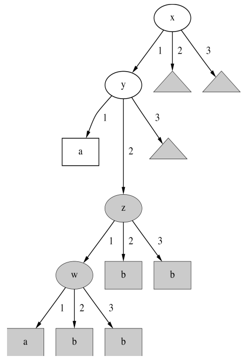



```
If x = 1 and y = 0
   then class = a
If x = 0 and y = 1
   then class = a
If x = 0 and y = 0
   then class = b
If x = 1 and y = 1
   then class = b
```



A tree with a replicated subtree

```
If x = 1 and y = 1
   then class = a
If z = 1 and w = 1
   then class = a
Otherwise class = b
```



"Nuggets" of knowledge

- Are rules independent pieces of knowledge? (It seems easy to add a rule to an existing rule base.)
- Problem: ignores how rules are executed
- Two ways of executing a rule set:
  - Ordered set of rules ("decision list")
    - Order is important for interpretation
  - Unordered set of rules
    - Rules may overlap and lead to different conclusions for the same instance

Interpreting rules

- What if two or more rules conflict?
  - Give no conclusion at all?
  - Go with rule that is most popular on training data?
  - …
- What if no rule applies to a test instance?
  - Give no conclusion at all?
  - Go with class that is most frequent in training data?
  - …

Special case: boolean class

- Assumption: if instance does not belong to class "yes", it belongs to class "no"
- Trick: only learn rules for class "yes" and use default rule for "no"

```
If x = 1 and y = 1 then class = a
If z = 1 and w = 1 then class = a
Otherwise class = b
```

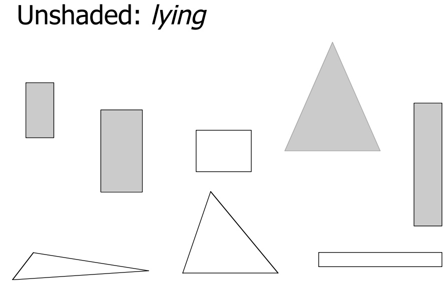
- Order of rules is not important. No conflicts!
- Rule can be written in *disjunctive normal form*

Rules involving relations

- So far: all rules involved comparing an attribute-value to a constant (e.g. temperature < 45)
- These rules are called "propositional" because they have the same expressive power as propositional logic
- What if problem involves relationships between examples (e.g. family tree problem from above)?
  - Can't be expressed with propositional rules
  - More expressive representation required

The shapes problem

- Target concept: *standing up*
- Shaded: *standing*
  Unshaded: *lying*



A propositional solution

Width	Height	Sides	Class
2	4	4	Standing
3	6	4	Standing
4	3	4	Lying
7	8	3	Standing
7	6	3	Lying
2	9	4	Standing
9	1	4	Lying
10	2	3	Lying

```
If width ≥ 3.5 and height < 7.0
   then lying
If height ≥ 3.5 then standing
```


A relational solution

- Comparing attributes with each other

```
If width > height then lying
If height > width then standing
```

- Generalizes better to new data
- Standard relations: =, <, >
- But: learning relational rules is costly
- Simple solution: add extra attributes (e.g. a binary attribute *is width < height?*)

Rules with variables

- Using variables and multiple relations:

```
If height_and_width_of(x,h,w) and h > w
   then standing(x)
```

- The top of a tower of blocks is standing:

```
If height_and_width_of(x,h,w) and h > w
     and is_top_of(x,y)
   then standing(x)
```

- The whole tower is standing:

```
If height_and_width_of(z,h,w) and h > w
     and is_top_of(x,z) and standing(y)
     and is_rest_of(x,y)
   then standing(x)
If empty(x) then standing(x)
```

- Recursive definition!

Inductive logic programming

- Recursive definition can be seen as logic program
- Techniques for learning logic programs stem from the area of "inductive logic programming" (ILP)
- But: recursive definitions are hard to learn
  - Also: few practical problems require recursion
  - Thus: many ILP techniques are restricted to non-recursive definitions to make learning easier

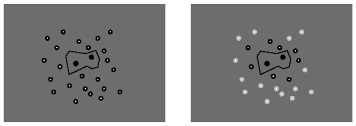
Instance-based representation

- Simplest form of learning: rote learning
  - Training instances are searched for instance that most closely resembles new instance
  - The instances themselves represent the knowledge
  - Also called instance-based learning
- Similarity function defines what's "learned"
- Instance-based learning is *lazy* learning
- Methods: k-nearest-neighbor, …

The distance function

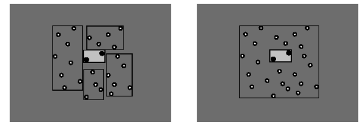
- Simplest case: one numeric attribute
  - Distance is the difference between the two attribute values involved (or a function thereof)
- Several numeric attributes: normally, Euclidean distance is used and attributes are normalized
- Nominal attributes: distance is set to 1 if values are different, 0 if they are equal
- Are all attributes equally important?
  - Weighting the attributes might be necessary

Learning prototypes



- Only those instances involved in a decision need to be stored
- Noisy instances should be filtered out
- Idea: only use *prototypical* examples

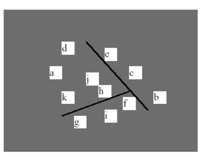
Rectangular generalizations



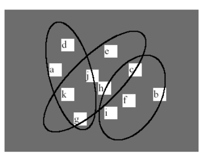
- Nearest-neighbor rule is used outside rectangles
- Rectangles are rules! (But they can be more conservative than "normal" rules.)
- Nested rectangles are rules with exceptions

Representing clusters I

Simple 2-D representation



Venn diagram



Overlapping clusters

Representing clusters II

Probabilistic assignment

	1	2	3
a	0.4	0.1	0.5
b	0.1	0.8	0.1
c	0.3	0.3	0.4
d	0.1	0.1	0.8
e	0.4	0.2	0.4
f	0.1	0.4	0.5
g	0.7	0.2	0.1
h	0.5	0.4	0.1

Dendrogram

NB: dendron is the Greek word for tree

Summary

- Trees
- Rules
- Relational representation
- Instance-based representation