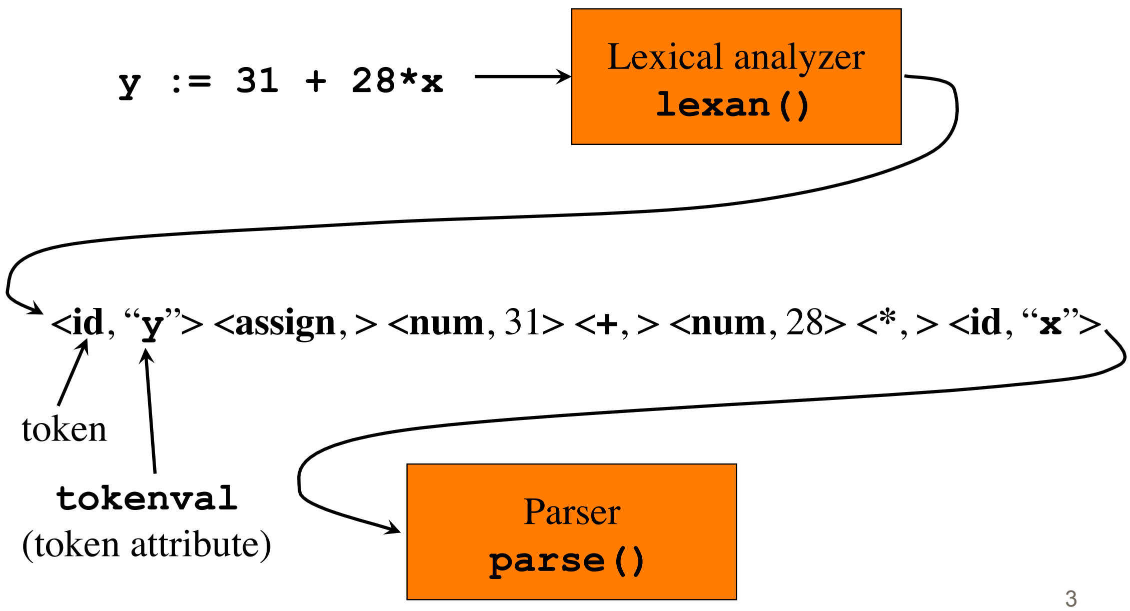
Lexical Analysis

Dr. Ir. Bart Kienhuis
Computer Systems Group
LIACS

Adding a Lexical Analyzer

- Typical tasks of the lexical analyzer:
  - Remove white space and comments
  - Encode constants as tokens
  - Recognize keywords
  - Recognize identifiers

The Lexical Analyzer



Token Attributes

factor → (*expr*)
| **num** { print(**num**.value) }

```
#define NUM 256 /* token returned by lexan */

factor()
{    if (lookahead == '(')
     {    match('('); expr(); match(')');
     }
     else if (lookahead == NUM)
     {    printf(" %d ", tokenval); match(NUM);
     }
     else error();
}
```

Symbol Table

The symbol table is globally accessible (to all phases of the compiler)

Each entry in the symbol table contains a string and a token value:

```
struct entry
{   char *lexptr; /* lexeme (string) */
    int token;
};
struct entry symtable[];
```

`insert(s, t)`: returns array index to new entry for string **`s`** token **`t`**

`lookup(s)`: returns array index to entry for string **`s`** or 0

Possible implementations:
- simple C code (see textbook)
- hashtables

Identifiers

factor → (*expr*)
| **id** { print(**id**.string) }

```
#define ID 259 /* token returned by lexan() */

factor()
{   if (lookahead == '(')
    {   match('('); expr(); match(')');
    }
    else if (lookahead == ID)
    {   printf(" %s ", symtable[tokenval].lexptr);
        match(NUM);
    }
    else error();
}
```

Handling Reserved Keywords

We simply initialize the global symbol table with the set of keywords

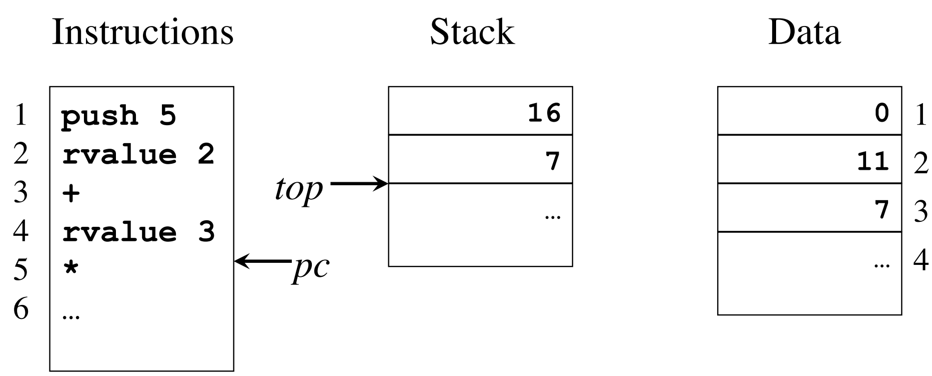
```
/* global.h */
#define DIV 257 /* token */
#define MOD 258 /* token */
#define ID  259 /* token */

/* init.c */
insert("div", DIV);
insert("mod", MOD);

/* lexer.c */
int lexan()
{   …
    tokenval = lookup(lexbuf);
    if (tokenval == 0)
        tokenval = insert(lexbuf, ID);
    return symtable[p].token;
}
```


Handling Reserved Keywords (cont'd)

morefactors → **div** *factor* { print('DIV') } *morefactors*
| **mod** *factor* { print('MOD') } *morefactors*
| …

```
/* parser.c */
morefactors()
{   if (lookahead == DIV)
    {   match(DIV); factor(); printf("DIV"); morefactors();
    }
    else if (lookahead == MOD)
    {   match(MOD); factor(); printf("MOD"); morefactors();
    }
    else
        …
}
```

Abstract Stack Machines



Generic Instructions for Stack Manipulation

push *v*	push constant value *v* onto the stack
rvalue *l*	push contents of data location *l*
lvalue *l*	push address of data location *l*
pop	discard value on top of the stack
:=	the r-value on top is placed in the l-value below it and both are popped
copy	push a copy of the top value on the stack
+	add value on top with value below it pop both and push result
-	subtract value on top from value below it pop both and push result
***, /,** …	ditto for other arithmetic operations
<, &, …	ditto for relational and logical operations

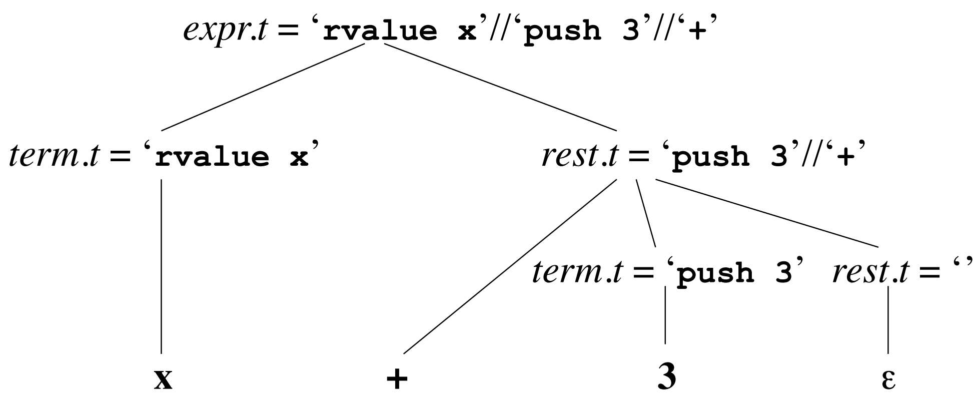
Generic Control Flow Instructions

`label` *l*	label instruction with *l*
`goto` *l*	jump to instruction labeled *l*
`gofalse` *l*	pop the top value, if zero then jump to *l*
`gotrue` *l*	pop the top value, if nonzero then jump to *l*
`halt`	stop execution
`jsr` *l*	jump to subroutine labeled *l*, push return address
`return`	pop return address and return to caller

Syntax-Directed Translation of Expressions

expr → *term rest* { *expr.t* := *term.t* // *rest.t* }
rest → **+** *term* $rest_1$ { *rest.t* := *term.t* // '+' // $rest_1.t$ }
rest → **-** *term* $rest_1$ { *rest.t* := *term.t* // '-' // $rest_1.t$ }
rest → ε { *rest.t* := '' }
term → **num** { *term.t* := '**`push`** ' // **num**.*value* }
term → **id** { *term.t* := '**`rvalue`** ' // **id**.*lexeme* }

Syntax-Directed Translation of Expressions (cont'd)



Translation Scheme to Generate Abstract Machine Code

expr → *term moreterms*
moreterms → **+** *term* { print('`+`') } *moreterms*
moreterms → **-** *term* { print('`-`') } *moreterms*
moreterms → ε
term → *factor morefactors*
morefactors → * *factor* { print('`*`') } *morefactors*
morefactors → **div** *factor* { print('`DIV`') } *morefactors*
morefactors → **mod** *factor* { print('`MOD`') } *morefactors*
morefactors → ε
factor → **(** *expr* **)**
factor → **num** { print('`push `' // **num**.*value*) }
factor → **id** { print('`rvalue `' // **id**.*lexeme*) }

Translation Scheme to Generate Abstract Machine Code (cont'd)

stmt → **id** **:=** { print('`lvalue `' // **id**.*lexeme*) } *expr* { print('`:=`') }

`lvalue` **id**.*lexeme*
code for *expr*
`:=`

Translation Scheme to Generate Abstract Machine Code (cont'd)

stmt → **if** *expr* { *out* := newlabel(); print('`gofalse `' // *out*) }
then *stmt* { print('`label `'// *out*) }

code for *expr*
`gofalse` *out*
code for *stmt*
`label` *out*

Translation Scheme to Generate Abstract Machine Code (cont'd)

stmt → **while** { *test* := newlabel(); print('**`label`** ' // *test*) }
expr { *out* := newlabel(); print('**`gofalse`** ' // *out*) }
do *stmt* { print('**`goto`** ' // *test* // '**`label`** ' // *out*) }

`label` *test*
code for *expr*
`gofalse` *out*
code for *stmt*
`goto` *test*
`label` *out*

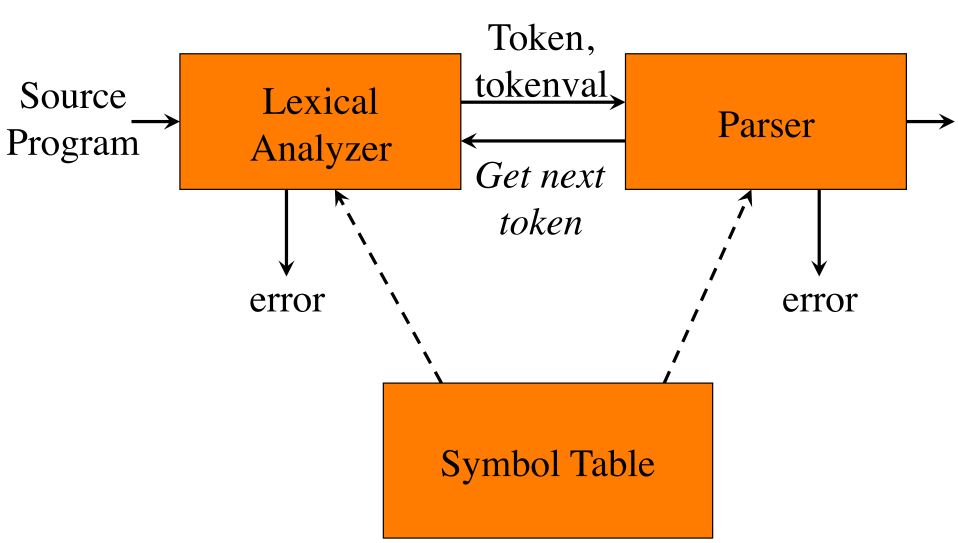
Translation Scheme to Generate Abstract Machine Code (cont'd)

start → *stmt* { print('**`halt`**') }
stmt → **begin** *opt_stmts* **end**
opt_stmts → *stmt* **;** *opt_stmts* | ε

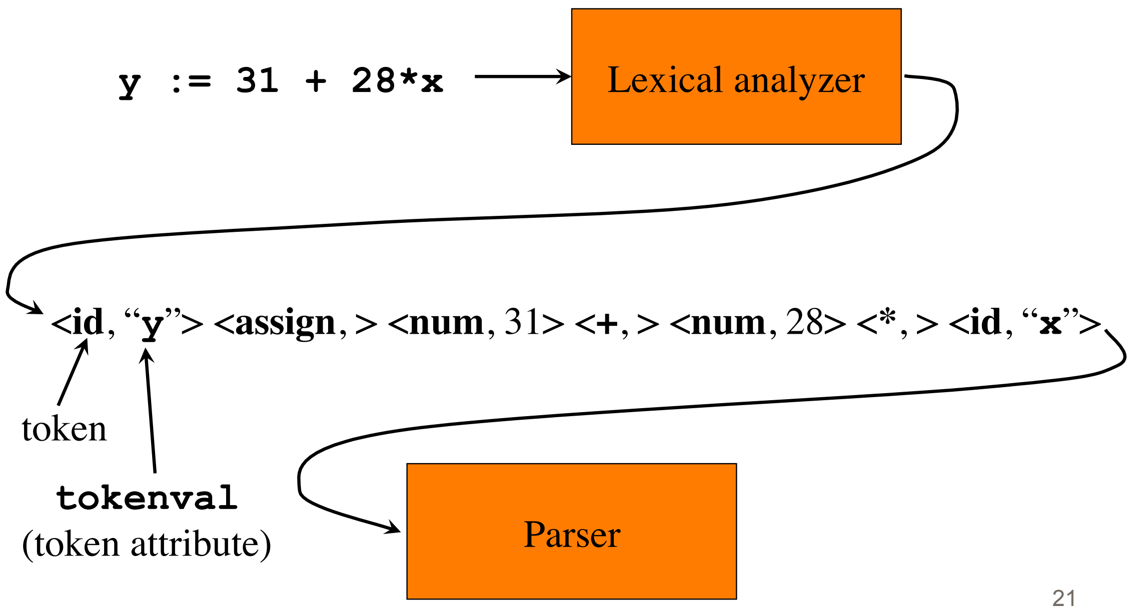
The Reason Why Lexical Analysis is a Separate Phase

- Simplifies the design of the compiler
- Provides efficient implementation
  - Systematic techniques to implement lexical analyzers by hand or automatically
  - Stream buffering methods to scan input
- Improves portability
  - Non-standard symbols and alternate character encodings can be more easily translated

Interaction of the Lexical Analyzer with the Parser



Attributes of Tokens



Tokens, Patterns, and Lexemes

- A *token* is a classification of lexical units
  - For example: **id** and **num**
- *Lexemes* are the specific character strings that make up a token
  - For example: `abc` and `123`
- *Patterns* are rules describing the set of lexemes belonging to a token
  - For example: "*letter followed by letters and digits*" and "*non-empty sequence of digits*"

Specification of Patterns for Tokens: Terminology

- An *alphabet* Σ is a finite set of symbols (characters)
- A *string* *s* is a finite sequence of symbols from Σ
  - $|s|$ denotes the length of string *s*
  - ε denotes the empty string, thus $|\varepsilon| = 0$
- A *language* is a specific set of strings over some fixed alphabet Σ

Specification of Patterns for Tokens: String Operations

- The *concatenation* of two strings *x* and *y* is denoted by *xy*
- The *exponentation* of a string *s* is defined by

  $s^0 = \varepsilon$

  $s^i = s^{i-1}s$ for $i > 0$

  (note that $s\varepsilon = \varepsilon s = s$)

Specification of Patterns for Tokens: Language Operations

- *Union*
  $L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
- *Concatenation*
  $LM = \{xy \mid x \in L \text{ and } y \in M\}$
- *Exponentiation*
  $L^0 = \{\varepsilon\};\ L^i = L^{i-1}L$
- *Kleene closure*
  $L^* = \cup_{i=0,\ldots,\infty} L^i$
- *Positive closure*
  $L^+ = \cup_{i=1,\ldots,\infty} L^i$

Specification of Patterns for Tokens: Regular Expressions

- Basis symbols:
  - ε is a regular expression denoting language $\{\varepsilon\}$
  - $a \in \Sigma$ is a regular expression denoting $\{a\}$
- If r and s are regular expressions denoting languages $L(r)$ and $M(s)$ respectively, then
  - $r \mid s$ is a regular expression denoting $L(r) \cup M(s)$
  - rs is a regular expression denoting $L(r)M(s)$
  - r^* is a regular expression denoting $L(r)^*$
  - (r) is a regular expression denoting $L(r)$
- A language defined by a regular expression is called a *regular set*

Specification of Patterns for Tokens: Regular Definitions

- Naming convention for regular expressions:

  $d_1 \to r_1$
  $d_2 \to r_2$
  …
  $d_n \to r_n$

  where r_i is a regular expression over
  $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$
- Each d_j in r_i is textually substituted in r_i

Specification of Patterns for Tokens: Regular Definitions

- Example:

  letter → `A` | `B` | … | `Z` | `a` | `b` | … | `z`
  digit → `0` | `1` | … | `9`
  id → **letter** (**letter** | **digit**)*

- Cannot use recursion, this is illegal:

  digits → **digit digits** | **digit**

Specification of Patterns for Tokens: Notational Shorthands

- We frequently use the following shorthands:

  $r^+ = rr^*$

  $r? = r \mid \varepsilon$

  `[a-z]` = `a` | `b` | `c` | … | `z`
- For example:

  digit → `[0-9]`

  num → **digit**$^+$ (**.** **digit**$^+$)? (**E** (**+**|**-**)? **digit** $^+$)?

Regular Definitions and Grammars

Grammar

stmt → **if** *expr* **then** *stmt*
  | **if** *expr* **then** *stmt* **else** *stmt*
  | ε

expr → *term* **relop** *term*
  | *term*

term → **id**
  | **num**

Regular definitions

if → `if`

then → `then`

else → `else`

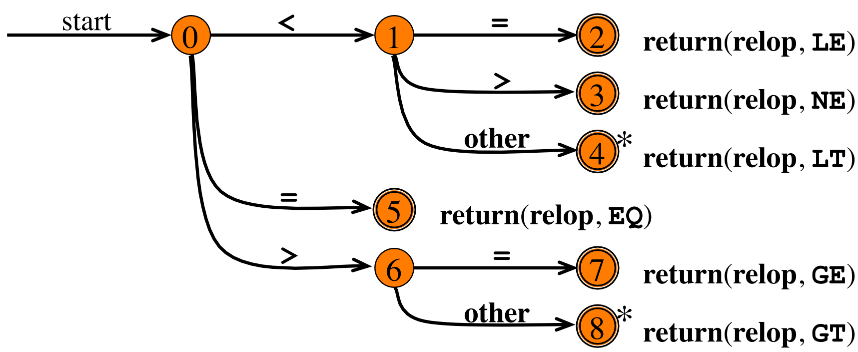
relop → `<` | `<=` | `<>` | `>` | `>=` | `=`

id → **letter** (**letter** | **digit**)*

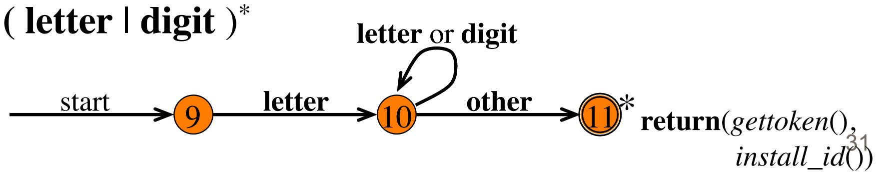
num → **digit**$^+$ (**.** **digit**$^+$)? (**E** (**+**|**-**)? **digit**$^+$)?

Implementing a Scanner Using Transition Diagrams

relop → < | <= | <> | > | >= | =



id → letter (letter | digit)*



Implementing a Scanner Using Transition Diagrams (Code)

```
token nexttoken()
{ while (1) {
    switch (state) {
    case 0: c = nextchar();
       if (c==blank || c==tab || c==newline) {
         state = 0;
         lexeme_beginning++;
       }
       else if (c=='<') state = 1;
       else if (c=='=') state = 5;
       else if (c=='>') state = 6;
       else state = fail();
       break;
     case 1:
       …
     case 9: c = nextchar();
       if (isletter(c)) state = 10;
       else state = fail();
       break;
     case 10: c = nextchar();
       if (isletter(c)) state = 10;
       else if (isdigit(c)) state = 10;
       else state = 11;
       break;
     …
```

Decides what other start state is applicable

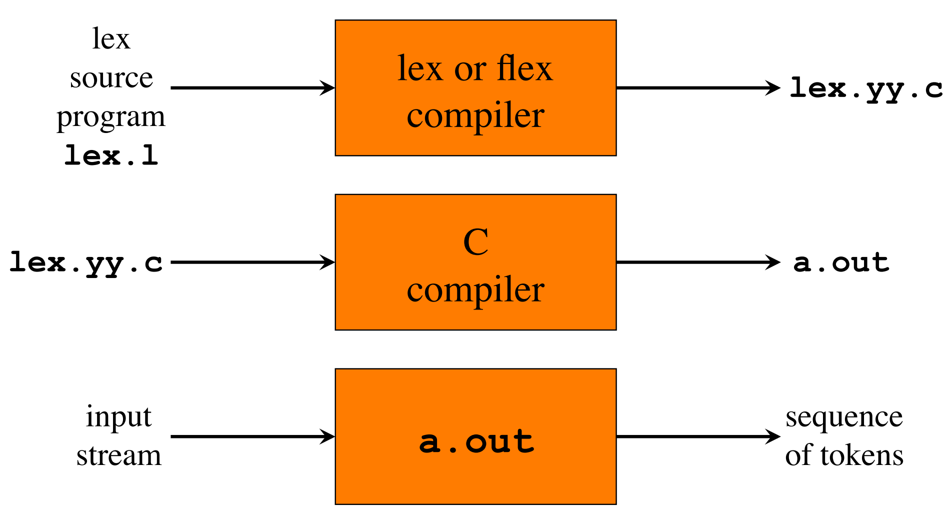
↓

```
int fail()
{ forward = token_beginning;
  swith (start) {
  case  0: start =  9; break;
  case  9: start = 12; break;
  case 12: start = 20; break;
  case 20: start = 25; break;
  case 25: recover(); break;
  default: /* error */
  }
  return start;
}
```

The Lex and Flex Scanner Generators

- *Lex* and its newer cousin *flex* are scanner generators
- Systematically translate regular definitions into C source code for efficient scanning
- Generated code is easy to integrate in C applications

Creating a Lexical Analyzer with Lex and Flex



Lex Specification

- A *lex specification* consists of three parts:
  regular definitions, C declarations in `%{ %}`
  `%%`
  translation rules
  `%%`
  user-defined auxiliary procedures
- The *translation rules* are of the form:
  p_1 { $action_1$ }
  p_2 { $action_2$ }
  …
  p_n { $action_n$ }

Regular Expressions in Lex

x	match the character **x**
\\.	match the character **.**
"*string*"	match contents of string of characters
.	match any character except newline
^	match beginning of a line
$	match the end of a line
[xyz]	match one character **x**, **y**, or **z** (use **** to escape **-**)
[^xyz]	match any character except **x**, **y**, and **z**
[a-z]	match one of **a** to **z**
r*****	closure (match zero or more occurrences)
r**+**	positive closure (match one or more occurrences)
r**?**	optional (match zero or one occurrence)
$r_1 r_2$	match r_1 then r_2 (concatenation)
r_1 **|** r_2	match r_1 or r_2 (union)
(r **)**	grouping
r_1****r_2	match r_1 when followed by r_2
{d**}**	match the regular expression defined by d

Example Lex Specification 1

Translation rules

```
%{
#include <stdio.h>
%}
%%
[0-9]+   { printf("%s\n", yytext); }
.|\n     { }
%%
main()
{ yylex();
}
```

Contains the matching lexeme

Invokes the lexical analyzer

```
lex spec.l
gcc lex.yy.c -ll
./a.out < spec.l
```


Example Lex Specification 2

Translation rules

```
%{
#include <stdio.h>
int ch = 0, wd = 0, nl = 0;
%}
delim      [ \t]+
%%
\n         { ch++; wd++; nl++; }
^{delim}   { ch+=yyleng; }
{delim}    { ch+=yyleng; wd++; }
.          { ch++; }
%%
main()
{ yylex();
  printf("%8d%8d%8d\n", nl, wd, ch);
}
```

Regular definition

Example Lex Specification 3

Translation rules

Regular definitions

```
%{
#include <stdio.h>
%}
digit      [0-9]
letter     [A-Za-z]
id         {letter}({letter}|{digit})*
%%
{digit}+   { printf("number: %s\n", yytext); }
{id}       { printf("ident: %s\n", yytext); }
.          { printf("other: %s\n", yytext); }
%%
main()
{ yylex();
}
```


Example Lex Specification 4

Return token to parser

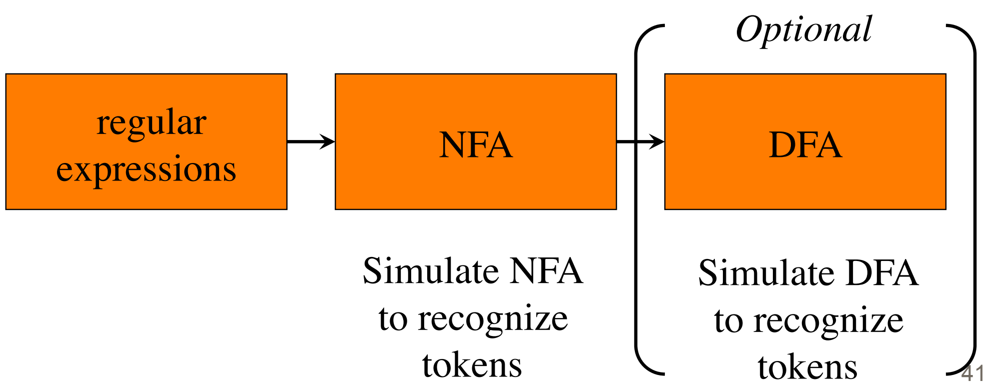
Token attribute

Install **yytext** as identifier in symbol table

```
%{ /* definitions of manifest constants */
#define LT (256)
…
%}
delim      [ \t\n]
ws         {delim}+
letter     [A-Za-z]
digit      [0-9]
id         {letter}({letter}|{digit})*
number     {digit}+(\.{digit}+)?(E[+\-]?{digit}+)?
%%
{ws}       { }
if         {return IF;}
then       {return THEN;}
else       {return ELSE;}
{id}       {yylval = install_id(); return ID;}
{number}   {yylval = install_num(); return NUMBER;}
"<"        {yylval = LT; return RELOP;}
"<="       {yylval = LE; return RELOP;}
"="        {yylval = EQ; return RELOP;}
"<>"       {yylval = NE; return RELOP;}
">"        {yylval = GT; return RELOP;}
">="       {yylval = GE; return RELOP;}
%%
int install_id()
…
```

Design of a Lexical Analyzer Generator

- Translate regular expressions to NFA
- Translate NFA to an efficient DFA



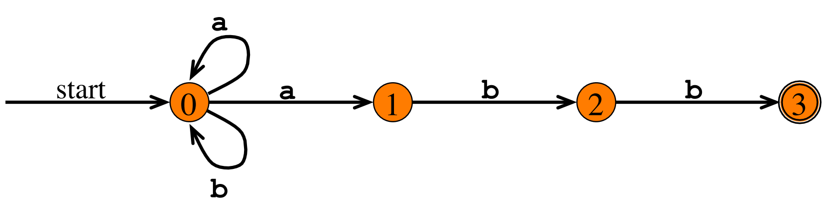
Nondeterministic Finite Automata

- Definition: an NFA is a 5-tuple (S,Σ,δ,s_0,F) where

  S is a finite set of *states*
  Σ is a finite set of *input symbol alphabet*
  δ is a *mapping* from $S\times\Sigma$ to a set of states
  $s_0 \in S$ is the *start state*
  $F \subseteq S$ is the set of *accepting* (or *final*) *states*

Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*



$S = \{0,1,2,3\}$
$\Sigma = \{\mathbf{a},\mathbf{b}\}$
$s_0 = 0$
$F = \{3\}$

Transition Table

- The mapping δ of an NFA can be represented in a *transition table*

$\delta(0,\mathbf{a}) = \{0,1\}$
$\delta(0,\mathbf{b}) = \{0\}$
$\delta(1,\mathbf{b}) = \{2\}$
$\delta(2,\mathbf{b}) = \{3\}$

→

State	*Input* **a**	*Input* **b**
0	{0, 1}	{0}
1		{2}
2		{3}

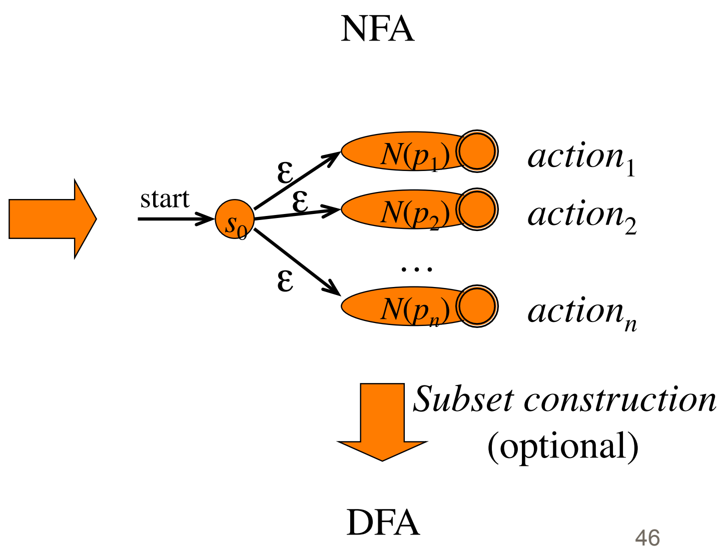
The Language Defined by an NFA

- An NFA *accepts* an input string *x* **iff** there is some path with edges labeled with symbols from *x* in sequence from the start state to some accepting state in the transition graph
- A state transition from one state to another on the path is called a *move*
- The *language defined by* an NFA is the set of input strings it accepts, such as (**a**|**b**)***abb** for the example NFA

Design of a Lexical Analyzer Generator: RE to NFA to DFA

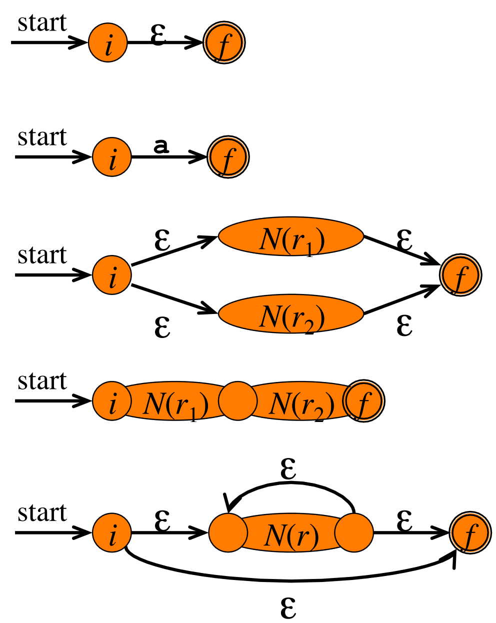
Lex specification with regular expressions

p_1	{ $action_1$ }
p_2	{ $action_2$ }
…	
p_n	{ $action_n$ }



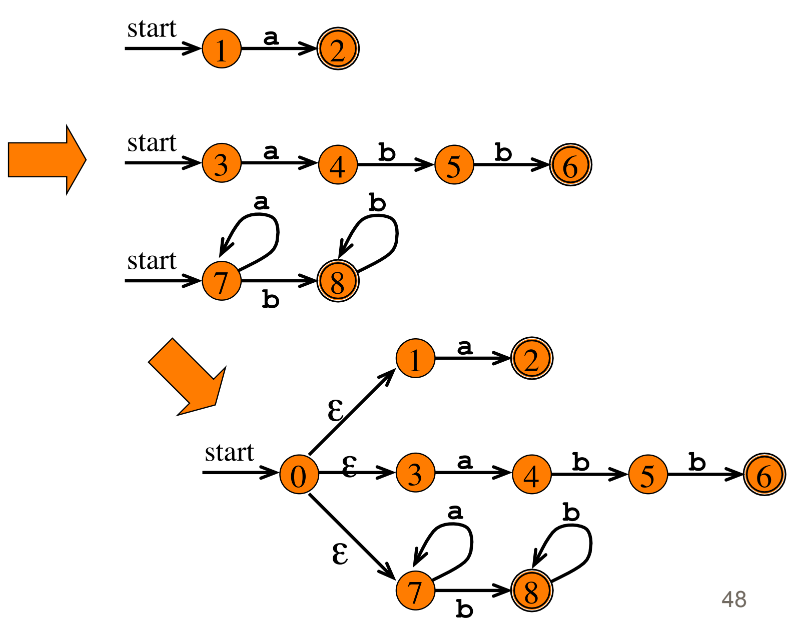
From Regular Expression to NFA (Thompson's Construction)

ε	
a	
$r_1 \mid r_2$	
$r_1 r_2$	
r^*	

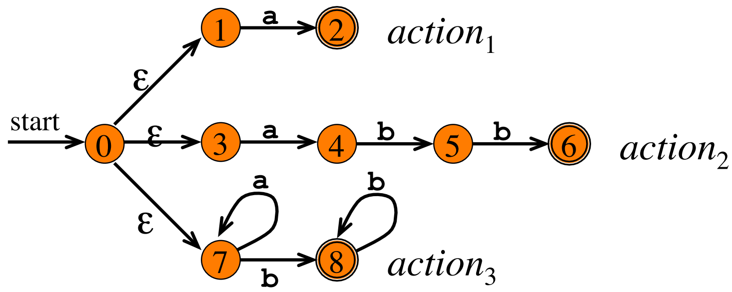


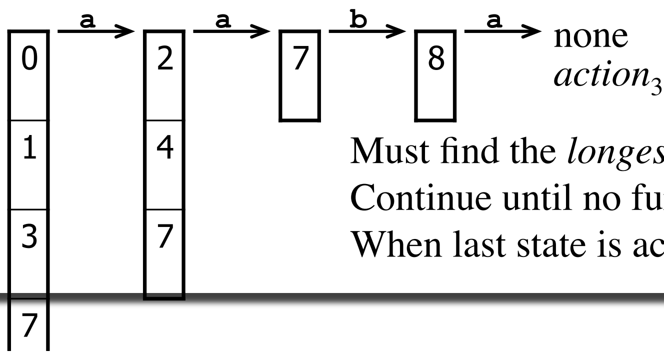
Combining the NFAs of a Set of Regular Expressions

a	{ $action_1$ }
abb	{ $action_2$ }
a*b+	{ $action_3$ }



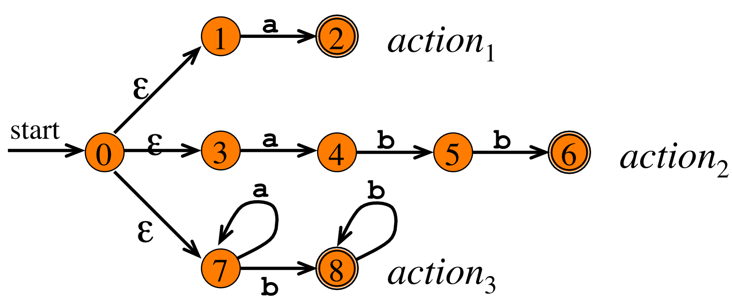
Simulating the Combined NFA Example 1

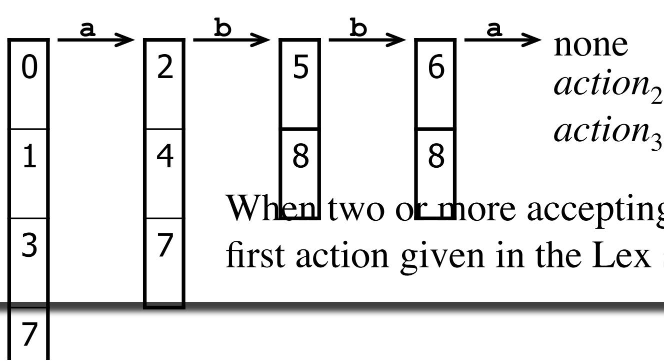




Must find the *longest match*:
Continue until no further moves are possible
When last state is accepting: execute action

Simulating the Combined NFA Example 2





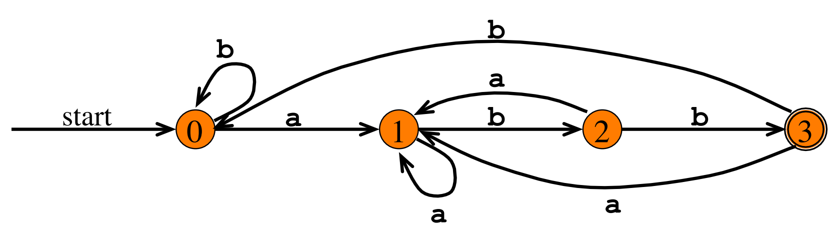
When two or more accepting states are reached, the first action given in the Lex specification is executed

Deterministic Finite Automata

- A *deterministic finite automaton* is a special case of an NFA
  - No state has an ε-transition
  - For each state *s* and input symbol *a* there is at most one edge labeled *a* leaving *s*
- Each entry in the transition table is a single state
  - At most one path exists to accept a string
  - Simulation algorithm is simple

Example DFA

A DFA that accepts (**a**|**b**)***abb**



Conversion of an NFA into a DFA

- The *subset construction algorithm* converts an NFA into a DFA using:

  $\varepsilon\text{-}closure(s) = \{s\} \cup \{t \mid s \to_\varepsilon \ldots \to_\varepsilon t\}$

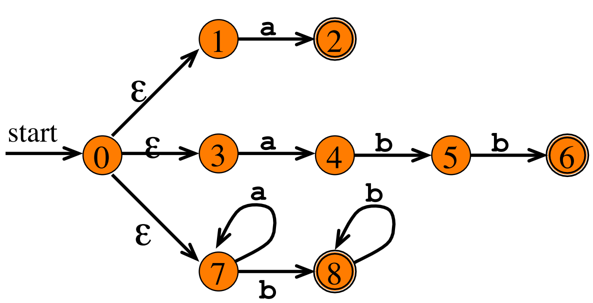
  $\varepsilon\text{-}closure(T) = \cup_{s \in T}\, \varepsilon\text{-}closure(s)$

  $move(T,a) = \{t \mid s \to_a t \text{ and } s \in T\}$
- The algorithm produces:

  Dstates is the set of states of the new DFA consisting of sets of states of the NFA

  Dtran is the transition table of the new DFA

ε-*closure* and *move* Examples



$\varepsilon\text{-}closure(\{0\}) = \{0,1,3,7\}$

$move(\{0,1,3,7\},\mathbf{a}) = \{2,4,7\}$

$\varepsilon\text{-}closure(\{2,4,7\}) = \{2,4,7\}$

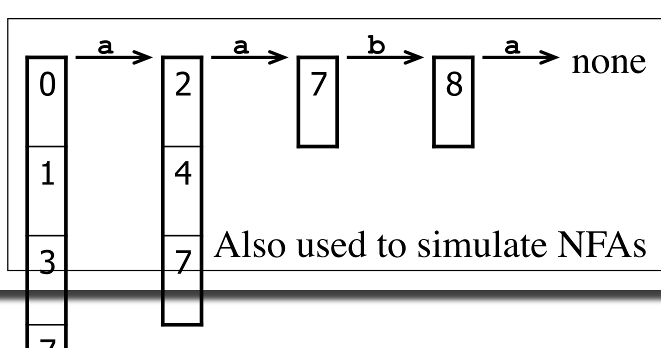
$move(\{2,4,7\},\mathbf{a}) = \{7\}$

$\varepsilon\text{-}closure(\{7\}) = \{7\}$

$move(\{7\},\mathbf{b}) = \{8\}$

$\varepsilon\text{-}closure(\{8\}) = \{8\}$

$move(\{8\},\mathbf{a}) = \emptyset$



Also used to simulate NFAs

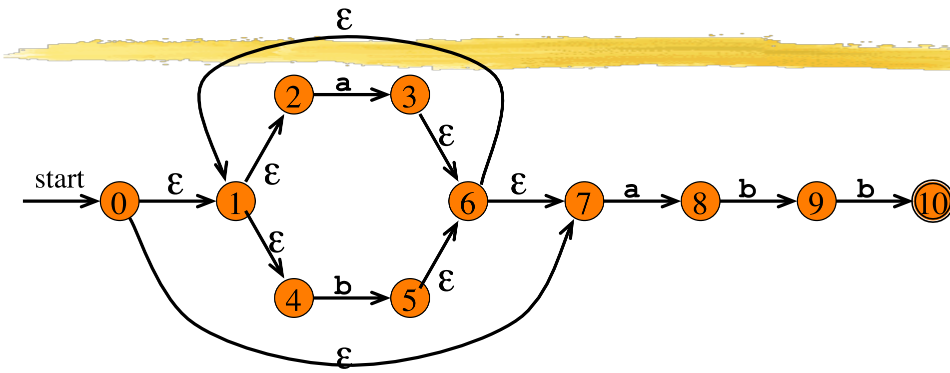
Simulating an NFA using ε-*closure* and *move*

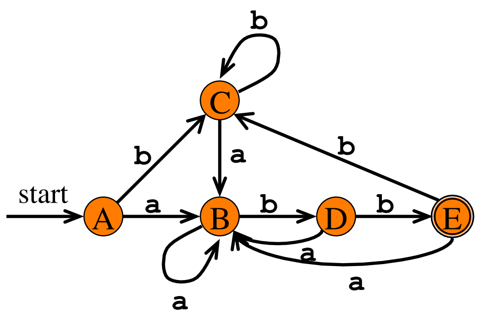
S := ε-*closure*({s_0})
S_{prev} := ∅
a := *nextchar*()
while $S \neq \emptyset$ **do**
    S_{prev} := S
    S := ε-*closure*(*move*(S,a))
    a := *nextchar*()
end do
if $S_{prev} \cap F \neq \emptyset$ **then**
    execute *action in* S_{prev}
    return "yes"
else **return** "no"

The Subset Construction Algorithm

Initially, ε-*closure*(s_0) is the only state in *Dstates* and it is unmarked
while there is an unmarked state T in *Dstates* **do**
    mark T
    for each input symbol $a \in \Sigma$ **do**
        U := ε-*closure*(*move*(T,a))
        if U is not in *Dstates* **then**
            add U as an unmarked state to *Dstates*
        end if
        Dtran[T,a] := U
    end do
end do

Subset Construction Example 1





Dstates

A = {0,1,2,4,7}

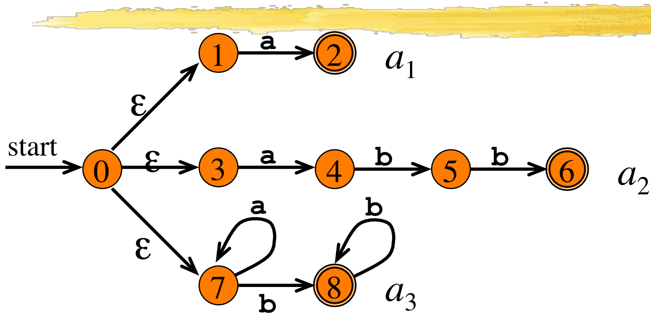
B = {1,2,3,4,6,7,8}

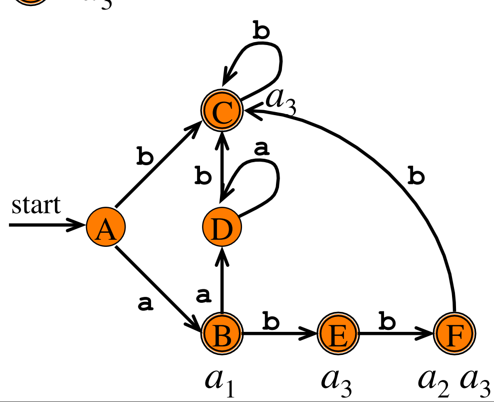
C = {1,2,4,5,6,7}

D = {1,2,4,5,6,7,9}

E = {1,2,4,5,6,7,10}

Subset Construction Example 2





Dstates

A = {0,1,3,7}

B = {2,4,7}

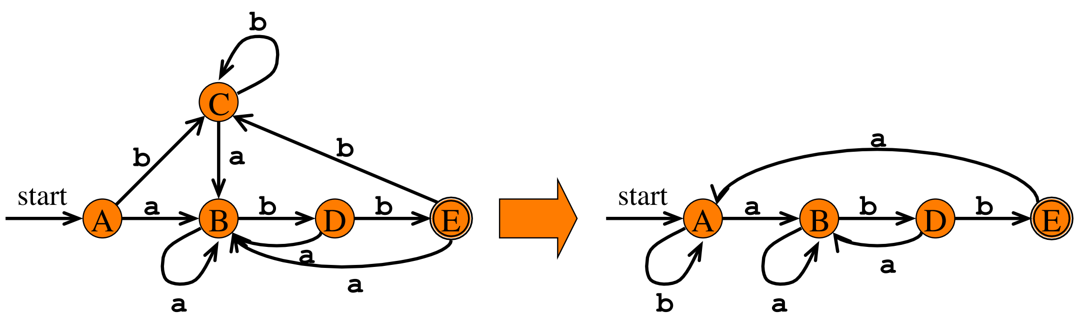
C = {8}

D = {7}

E = {5,8}

F = {6,8}

Minimizing the Number of States of a DFA



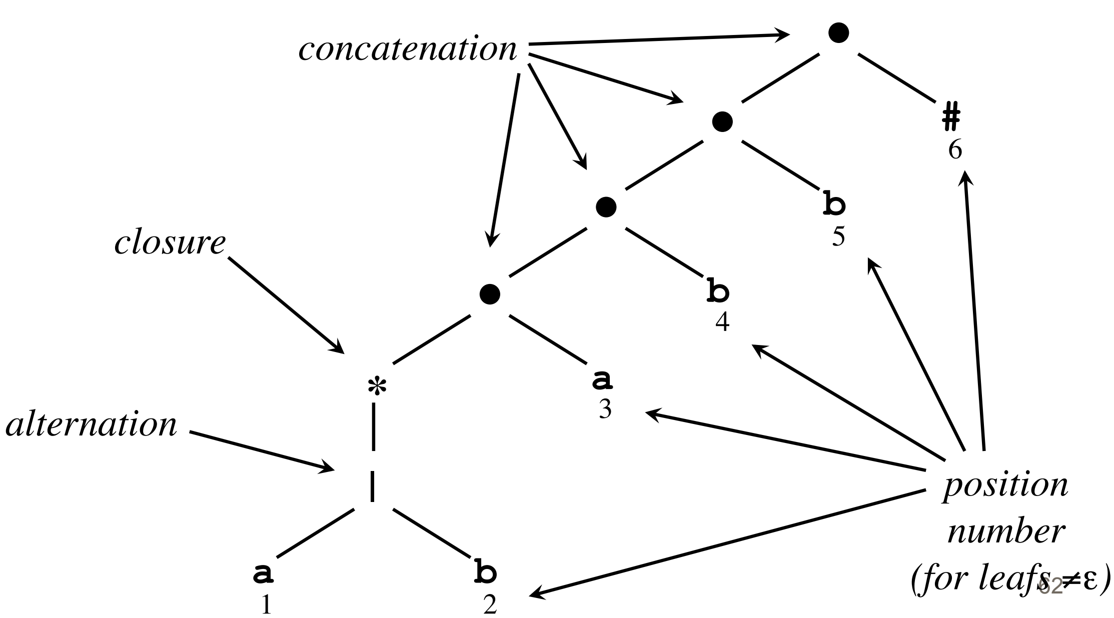
From Regular Expression to DFA Directly

- The *important states* of an NFA are those without an ε-transition, that is if *move*({*s*},*a*) ≠ ∅ for some *a* then *s* is an important state
- The subset construction algorithm uses only the important states when it determines ε-*closure*(*move*(*T*,*a*))

From Regular Expression to DFA Directly (Algorithm)

- Augment the regular expression *r* with a special end symbol # to make accepting states important: the new expression is *r#*
- Construct a syntax tree for *r#*
- Traverse the tree to construct functions *nullable*, *firstpos*, *lastpos*, and *followpos*

From Regular Expression to DFA Directly: Syntax Tree of (a|b)*abb#



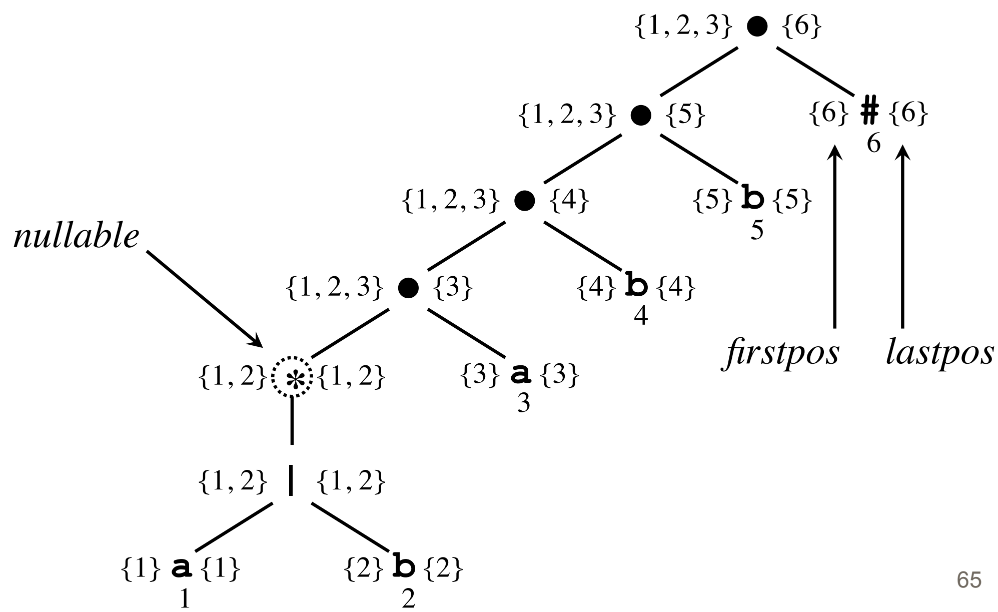
From Regular Expression to DFA Directly: Annotating the Tree

- *nullable*(*n*): the subtree at node *n* generates languages including the empty string
- *firstpos*(*n*): set of positions that can match the first symbol of a string generated by the subtree at node *n*
- *lastpos*(*n*): the set of positions that can match the last symbol of a string generated be the subtree at node *n*
- *followpos*(*i*): the set of positions that can follow position *i* in the tree

From Regular Expression to DFA Directly: Annotating the Tree

Node *n*	*nullable*(*n*)	*firstpos*(*n*)	*lastpos*(*n*)
Leaf ε	true	∅	∅
Leaf *i*	false	{*i*}	{*i*}
$\vert$ / \ c_1 c_2	*nullable*(c_1) or *nullable*(c_2)	*firstpos*(c_1) ∪ *firstpos*(c_2)	*lastpos*(c_1) ∪ *lastpos*(c_2)
• / \ c_1 c_2	*nullable*(c_1) and *nullable*(c_2)	**if** *nullable*(c_1) **then** *firstpos*(c_1) ∪ *firstpos*(c_2) **else** *firstpos*(c_1)	**if** *nullable*(c_2) **then** *lastpos*(c_1) ∪ *lastpos*(c_2) **else** *lastpos*(c_2)
* $\vert$ c_1	true	*firstpos*(c_1)	*lastpos*(c_1)

From Regular Expression to DFA Directly: Syntax Tree of (a|b)*abb#



From Regular Expression to DFA Directly: *followpos*

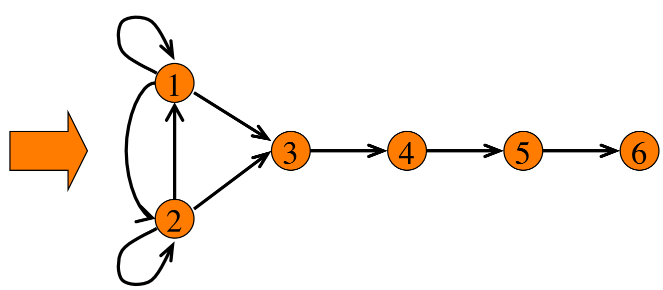
for each node n in the tree **do**
  if n is a cat-node with left child c_1 and right child c_2 **then**
    for each i in $lastpos(c_1)$ **do**
      $followpos(i) := followpos(i) \cup firstpos(c_2)$
    end do
  else if n is a star-node
    for each i in $lastpos(n)$ **do**
      $followpos(i) := followpos(i) \cup firstpos(n)$
    end do
  end if
end do

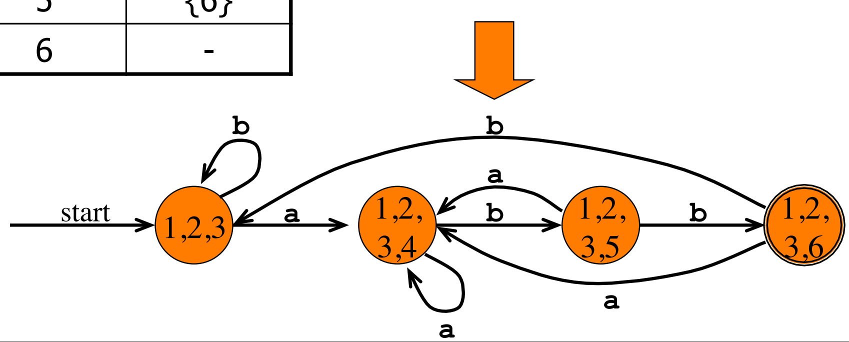
From Regular Expression to DFA Directly: Algorithm

s_0 := *firstpos*(*root*) where *root* is the root of the syntax tree
Dstates := $\{s_0\}$ and is unmarked
while there is an unmarked state T in *Dstates* **do**
    mark T
    for each input symbol $a \in \Sigma$ **do**
        let U be the set of positions that are in *followpos*(p)
            for some position p in T,
            such that the symbol at position p is a
        if U is not empty and not in *Dstates* **then**
            add U as an unmarked state to *Dstates*
        end if
        Dtran[T,a] := U
    end do
end do

From Regular Expression to DFA Directly: Example

Node	*followpos*
1	{1, 2, 3}
2	{1, 2, 3}
3	{4}
4	{5}
5	{6}
6	-





Time-Space Tradeoffs

Automaton	*Space (worst case)*	*Time (worst case)*
NFA	$O(\|r\|)$	$O(\|r\| \times \|x\|)$
DFA	$O(2^{\|r\|})$	$O(\|x\|)$