Quantum Teleportation

Hardware

Lecture 10

Quantum Teleportation

- It is the process by which the state of an arbitrary qubit is transferred from one location to another.
- Not science fiction, it has been performed in the laboratory.
- No-cloning theorem: not possible to make a copy of the state of an arbitrary qubit → when the state of the original qubit is teleported to another location, the state of the original will necessarily destroyed. “Move is possible, copy is impossible.”

Some preliminaries

- Switching between a canonical and a noncanonical basis can be helpful (see B92 protocol).
- A single qubit
  - – Canonical basis $\{|0\rangle, |1\rangle\}$
  - – Noncanonical basis $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$

- The teleportation algorithm works with two entangled qubits, one held by Alice and one held by Bob.
- Obvious canonical basis for this 4-dimensional space

$$\{|0_A 0_B\rangle, |0_A 1_B\rangle, |1_A 0_B\rangle, |1_A 1_B\rangle\}$$

- A noncanonical basis, called the Bell basis, consists of

$$\left|\Psi^+\right\rangle = \frac{|0_A 1_B\rangle + |1_A 0_B\rangle}{\sqrt{2}}$$

$$\left|\Psi^-\right\rangle = \frac{|0_A 1_B\rangle - |1_A 0_B\rangle}{\sqrt{2}}$$

$$\left|\Phi^+\right\rangle = \frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$$

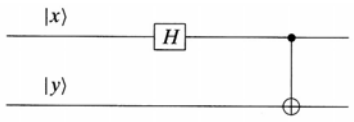
$$\left|\Phi^-\right\rangle = \frac{|0_A 0_B\rangle - |1_A 1_B\rangle}{\sqrt{2}}$$

- Every vector in this basis is entangled. See book for prove that it is indeed a basis.

- How are the Bell basis vectors formed?
- In the 2-dimensional case the elements of the noncanonical basis can be formed by the Hadamard matrix:

$$|0\rangle \to \frac{|0\rangle+|1\rangle}{\sqrt{2}} \quad \text{and} \quad |1\rangle \to \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

- In the 4-dimensional case:

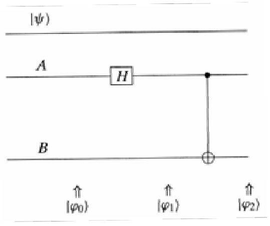


- It can be shown that this quantum circuit with appropriate inputs creates the elements of the Bell basis:

$$|00\rangle \to \left|\Phi^+\right\rangle, \quad |01\rangle \to \left|\Psi^+\right\rangle, \quad |10\rangle \to \left|\Phi^-\right\rangle, \quad |11\rangle \to \left|\Psi^-\right\rangle$$

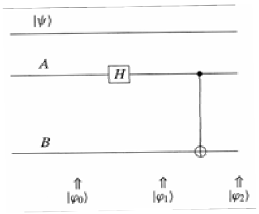
Quantum teleportation protocol

- Alice has qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ in an arbitrary state that she would like to teleport to Bob.
- **Step 1.** Two entangled qubits are formed as $\left|\Phi^+\right\rangle$. One is given to Alice and one is give to Bob. Three qubits as three lines:



- The top two lines are in Alice’s possession and the bottom line in Bob’s.

Step 1



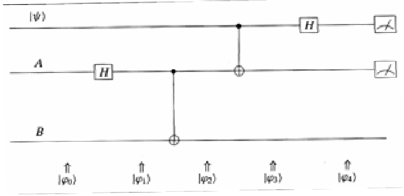
$$|\varphi_0\rangle = |\psi\rangle \otimes |0_A\rangle \otimes |0_B\rangle = |\psi\rangle \otimes |0_A 0_B\rangle,$$

$$|\varphi_1\rangle = |\psi\rangle \otimes \frac{|0_A\rangle + |1_A\rangle}{\sqrt{2}} \otimes |0_B\rangle,$$

$$|\varphi_2\rangle = |\psi\rangle \otimes |\Phi^+\rangle = |\psi\rangle \otimes \frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$$
$$= (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$$
$$= \frac{\alpha|0\rangle(|0_A 0_B\rangle + |1_A 1_B\rangle) + \beta|1\rangle(|0_A 0_B\rangle + |1_A 1_B\rangle)}{\sqrt{2}}$$

Step 2

- Alice lets her $|\psi\rangle$ interact with her entangled qubit.



- We have $|\varphi_3\rangle = \frac{\alpha|0\rangle(|0_A 0_B\rangle + |1_A 1_B\rangle) + \beta|1\rangle(|1_A 0_B\rangle + |0_A 1_B\rangle)}{\sqrt{2}}$

$$|\varphi_4\rangle = \frac{1}{2}\left(\alpha(|0\rangle + |1\rangle)(|0_A 0_B\rangle + |1_A 1_B\rangle) + \beta(|0\rangle - |1\rangle)(|1_A 0_B\rangle + |0_A 1_B\rangle)\right)$$
$$= \frac{1}{2}\left(\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \beta(|010\rangle + |001\rangle - |110\rangle - |101\rangle)\right)$$

Step 2 (cont'd)

- Regrouping these triplets $|xyz\rangle$ in terms of $|xy\rangle$, which is in Alice's possession

$$|\varphi_4\rangle = \frac{1}{2}\left(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\beta|0\rangle + \alpha|1\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(-\beta|0\rangle + \alpha|1\rangle)\right)$$

- So the system of three qubits is now in a superposition of four possible states.

Step 3

- Alice measures her two qubits and determines to which of the four possible states the system collapses.
- At the moment Alice measures her two qubits, all three qubits collapse to one of the four possibilities. So if she measures $|10\rangle$ then the third qubit is in state $\alpha|0\rangle - \beta|1\rangle$.
- Two problems:
  - Alice knows this state but Bob does not.
  - Bob has $\alpha|0\rangle - \beta|1\rangle$, not the desired $\alpha|0\rangle + \beta|1\rangle$
- Both problems are solved in Step 4.

Step 4

- Alice sends copies of her two bits (not qubits) to Bob who uses that information to achieve the desire state $|\psi\rangle$.
- E.g., if Bob receives $|10\rangle$ from Alice, he then knows that his qubits is in a state

$$\alpha|0\rangle - \beta|1\rangle = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$$

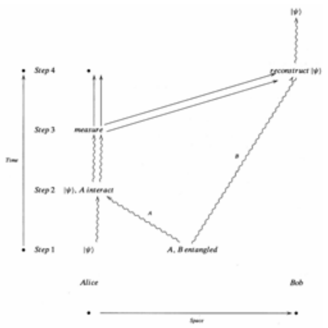
- Hence he should act on his qubit with the following matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ -\beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle + \beta|1\rangle = |\psi\rangle$$

- Bob must apply the following matrices

$$\begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{array}$$

- Space-time diagram, where straight arrows correspond to movement of bits and curvy arrows correspond to qubits on the move.



- Notice that $|\psi\rangle$ moves from the lower-left corner in Alice's possession to the upper-right corner in Bob's possession. Mission accomplished!

Remarks

- Alice is no longer in possession of the original state. She has only two classical bits.
- To “teleport” a single quantum particle, Alice has to send two classical bits. Without these Bob cannot know what he has. The classical bits travel via a classical channel (less than the speed of light). So entanglement does not allow you to communicate faster than the speed of light.
- α and β were arbitrary complex numbers. So they could have had an infinite decimal expansion. This potentially infinite amount of information goes from Alice to Bob via only two bits. However, it is passed as a qubit and useless to Bob. As soon as he measures the qubit, it will collapseto a bit.
- Is it teleportation? No particle has been moved at all! However, two particles having exactly the same quantum state are, from a standpoint of physics, indistinguishable and can therefore be treated as the same particle.

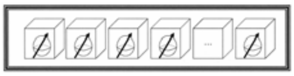
Hardware

- Do we actually know how to build a quantum computer?
- Formidable challenge to engineers and applied physicists
- Considering the amount of resources (academia, private sector, military) it would not be surprising if noticeable progress will be made in the near future.
- Disclaimer: area of research that requires a deep background in quantum physics and quantum engineering. Therefore a rather elementary discussion.

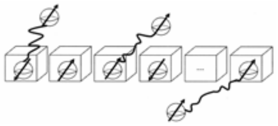
Goals and challenges

- Generic architecture:
  - Number of addressable qubits
  - Capable of initializing them properly
  - Apply a sequence of unitary transformations
  - Finally measuring them.

- Initialization: set machine in a well-defined state
  - Problem: entanglement between subsystems regardless their physical separation
  - Entanglement between machine and environment
    - Pure state



    - Mixed state



  - Problem:
    - No idea about the precise state of the environment’s electrons
    - No details of their interaction with the electrons in the quantum register.

Pure and mixed states

- What’s the difference?
- Consider the following family of spin states: $|\psi_\theta\rangle = \frac{|0\rangle + \exp(i\theta)|1\rangle}{\sqrt{2}}$
- For every choice of the angle θ, there is a distinct pure state.
- Each state is characterized by a specific relative phase (difference between angles of |0> and |1> in the polar representation).
- How can we detect their difference?
  - In standard basis will not work
  - A change of basis will do: the average spin value A along the x-axis depends on θ (see book): $A = \cos(\theta)$
  - Tossing a coin contains no relative phase → mixed state.
- The loss of purity of the state of a quantum system as the result of interaction with the environment is known as decoherence.

Decoherence

- We always implicitly assumed that we knew exactly how the environment affects the quantum system.
- More realistic scenario: a single electron is immersed in a vast environment, e.g., a single external electron.
- Electron has become entangled with another electron

$$|\psi_{\text{global}}\rangle = \frac{|00\rangle + \exp(i\theta)|11\rangle}{\sqrt{2}}$$

- What is the spin of our electron in the x-direction? 0 instead of a dependence on θ! (see book) It turns out that we should measure both electrons to get the dependence on θ.
- In general: we should measure all electrons of the environment. This is impossible, so our pure state is turned into a mixed one.
- Decoherence does not collapse the state vector: all information is still available!

Challenge due to decoherence

- On the one hand, adopting basic quantum systems that are very prone to “hook up” with the environment makes it difficult to manage the state of the machine.
- On the other hand, we do need to interact with the quantum device. Systems that tend to stay aloof makes it difficult to access their states.
- Can we hope to build a reliable quantum computing device if decoherence plays such an important role?
  - Fast gates execution: make decoherence sufficient slow compared to our control.
  - Fault-tolerance:
    - Quantum error-correcting codes
    - Repeat calculations

DiVincenzo’s wish list

1. The quantum machine must have a sufficient large number of individually addressable qubits.
2. It must be possible to initialize all the qubits to the zero state.
3. The error rate in doing computations should be reasonable low, i.e., decoherence time must be substantially longer than gate operation time.
4. We should be able to perform elementary logical operations between pairs of qubits.
5. Finally, we should be able to reliably read out the results of measurements.

Implementing a quantum computer

- A qubit is a state vector in a two-dimensional Hilbert space.
- Any physical quantum system whose state space has dimension 2^N can, in principle, be used to store an addressable sequence of N qubits.
- Options
  - Standard: quantum system with a two-dimensional state space.
  - Quantum register can be implemented by a number of copies.
  - Canonical two-dimensional quantum systems are particles with spin, e.g., electrons and single atoms.
  - Another choice is excited states of atoms.

Ion traps

- Oldest, most popular proposal
- Core idea: an ion is an electrically charged atom. Two types:
  - Positive ions or cations (lost one or more electrons)
  - Negative ions or anions (acquired some electrons)
- Ions can be acted upon by means of an electromagnetic field, or even better they can be confined in a specific volume, known as ion trap

- Practice: Ca$^+$

- How are qubits encoded? Ground state and exited state.

- Initialization:
  - Optical pumping: a laser pumps energy into an atom, that absorbs a photon, and raises from ground state to excited state. It can lose energy by emitting a photon.
  - Initialization of a register to some initial state possible with a high degree of fidelity (almost 100%).
- Manipulation:
  - Single-qubit rotation: by “hitting” the single ion with a laser pulse of a given amplitude, frequency, and duration, one can rotate its state appropriately.
  - Two-qubit gates: the ions in the trap are strung together by what is known as their common vibrational modes. A laser can affect their common mode, achieving the desired entanglement.
- Measurement:
  - Two main long-lived states |0> and |1>, and also a short-lived state |*s*> in the middle of |0>and |1>.
  - If ion is in ground state, gets pushed to |s>, it will revert to ground state and emits a photon. If it is in the excited state, it will not. Repeat this many times, and detect if photons are emitted to establish where the qubit is.

+ and – of ion trap

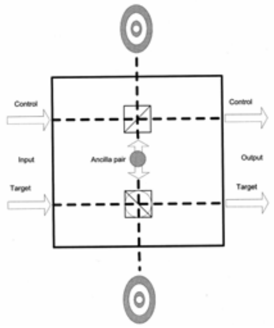
- On the plus side
  - Mode has a long coherence time, order 1-10s.
  - Measurements quite reliable, close to 100%.
  - Qubits can be transported around in the computer.
- On the minus side
  - Ion trap is slow in terms of gate time (order of 10ms)
  - Not apparent how to scale the optical part to thousands of qubits.

Linear optics

- Qubits:
  - Polarized photons
- Initialization:
  - Polarization filter
- Gates:
  - Nontrivial, since photons have a tendency to stay aloof
  - Implement some small universal set of quantum gates, e.g., controlled NOT gate
- Measurement:
  - Polarization filters and single-photon detectors.

Optical controlled-NOT gate

- Linear optics quantum computing (LOQC)
- LOQC-based controlled-NOT gate



+ and – of the optical scheme

- On the plus side:
  - Light *travels*. This means that quantum gates and quantum memory devices can be easily connected via optical fibers.
- On the minus side:
  - It is not easy for photons to become entangled. Also a plus wrt decoherence, but it makes gate creation challenging.

Nuclear Magnetic Resonance (NMR)

- Idea: encode qubits not as single particles or atoms, but as global spin states of many molecules in some fluid.
- These molecules float in a cup which is placed in an NMR machine.
- Contains plenty built-in redundancy → maintain coherence for a relatively long time span (several seconds).
- 1998: first two-qubit NMR computers.

Superconductor Quantum Computers (SQP)

- NMR uses fluids, SQP employs superconductors.
- By means of Josephson junctions – thin layers of nonconducting material sandwiched between two pieces of superconducting metal.
- At very low temperatures, electrons within a superconductor pair up to form a “superfluid” flowing with no resistance and as a single, uniform wave pattern.
- The current flows back and forth through the junction, like a ping-pong ball, in a rhythmic fashion.
- Implementation of qubits:
  - Through the Josephson junction qubit
  - The |0> and |1> states are represented by the two lowest-frequency oscillations of the currents.

Where are we now?

- In 2001 the first execution of Shor’s algorithm was carried out at IBM’s Almaden Research Center and Stanford University: 15 = 5 x 3!
- In 2005 a 12-bit NMR quantum register was benchmarked. Scalability seems to be a major hurdle.
- Recent news: NIST Road Map
  - NIST = US National Institute of Science and Technology
  - Major directions toward quantum hardware
  - http://qist.lanl.gov/qcomp_map.shtml
- Companies whose main business is developing Quantum Computing.

NIST Demonstrates 'Universal' Programmable Quantum Processor for Quantum Computers

Source: NIST news

Optical 'quantum computer chip' performs first calculation

According to Physics World researchers at the University of Bristol, UK have made a prototype optical quantum computer chip and used it to perform a mathematical calculation. The device consists of tiny silica waveguides on a silicon chip and carries out a version of the quantum calculation known as Shor's algorithm. The result is an important step towards making practical, real-world quantum computers.

Source: Physics World
Read more at:

Physicists set new entanglement record

Researchers in China and Europe have entangled a record-breaking 10 quantum bits — an important breakthrough in the quest to develop practical quantum computers.

Source: PhysicsWorld
Read more at:

Scaling Up a Quantum Computer

Researchers at the National Institute of Standards and Technology (NIST) in Boulder, CO, have demonstrated multiple computing operations on quantum bits—a crucial step toward building a practical quantum computer.

Source: MIT Technology Review
Read more at:

Atoms teleport information over long distance

Physicists have teleported quantum information between two atoms separated by a significant distance, for the first time. Until now this feat had only been achieved between photons, and between two nearby atoms through the intermediary action of a third. "Our system has the potential to form the basis for a large-scale 'quantum repeater' that can network quantum memories over vast distances" said group leader Christopher Monroe of the University of Maryland.

Source: physicsworld.com
Read more at:

Future of Quantum Ware

- Quantum computing may become a reality in the future, perhaps even in the relatively near future.
- Likely that many areas of information technology will be affected, in particular communication and cryptography.
- If sizeable quantum devices become available: impact of artificial intelligence.
- Science fiction…
- The dreams of today are the reality of tomorrow.

Exam

Date: Mon Jan 11th, 10-13h (not Jan 25th!)

Location: *to be determined*

Book: chapters 1, 2, 3, 4, 5, 6, 9 &11