Math background

The Leap from Classical to Quantum

Lecture 3

Eigenvalues and eigenvectors

- For a matrix A in $C^{m \times n}$, if there is a number c in C and a vector $V \neq 0$ within C^n such that $AV = c \cdot V$, then c is called an eigenvalue of A and V an eigenvector of A associated with c.
- Some matrices have many eigenvalues and eigenvectors and some matrices have none.

Eigenspace

- If A has eigenvalue c_0 with eigenvector V_0, then for any $c \in$ C we have

$$A(cV_0) = cAV_0 = cc_0V_0 = c_0(cV_0)$$

which shows that cV_0 is also an eigenvector of A with eigenvalue c_0.

- If cV_0 and $c'V_0$ are two such eigenvectors, then because of

$$A(cV_0 + c'V_0) = AcV_0 + A\,c'V_0 = cAV_0 + c'AV_0$$
$$= c(c_0V_0) + c'(c_0V_0) = (c + c')(c_0V_0) = c_0(c+c')\,V_0$$

we see that the addition of two such eigenvectors is also an eigenvector.

- Therefore, every eigenvector determines a complex subvector space of the vector space. It is known as the eigenspace associated with the given eigenvector.

Hermitian matrices

- An $n \times n$ matrix A is called hermitian if $A^\dagger = A$. In other words $A[j,k] = \overline{A[k,j]}$.
- If A is a hermitian matrix then the operator that it represents is called self-adjoint.
- If A is a hermitian $n \times n$ matrix, we have $<AV,V'> = <V,AV'>$.
- If A is hermitian, then all eigenvalues are real.
- For a given hermitian matrix, distinct eigenvectors that have distinct eigenvalues are orthogonal.
- A diagonal matrix is a square matrix whose only nonzero entries are on the diagonal. All entries off the diagonal are zero.
- Every self-adjoint operator A on a finite-dimensional complex vector space **V** can be represented by a diagonal matrix whose diagonal entries are the eigenvalues of A, and whose eigenvectors form an orthonormal basis for **V** (we call this basis an eigenbasis).
- With every physical observable of a quantum system there is a corresponding hermitian matrix. Measurements of the observable always leads to a state that is represented by one of the eigenvectors of the associated hermitian matrix.

Unitary matrices

- An $n \times n$ matrix U is called unitary if $U * U^\dagger = I_n$.
- Unitary matrices preserve inner products $<UV,UV'> = <V,V'>$.
- Unitary matrices preserve distances $d(UV_1,UV_2) = d(V_1,V_2)$. An operator that preserves distances is called an isometry.
- If U is unitary and $UV = V'$, then we can easily form $U^\dagger$ and by multiplying both sides we get $U^\dagger\, UV = U^\dagger V'$ or $V = U^\dagger V'$. In other words $U^\dagger$ can "undo" the action that U performs. In the quantum world all actions (that are not measurements) are "undoable" or "reversible".

Tensor product

- Most difficult, most essential subject!
- Tensor product of vectors

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \otimes \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_0 \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \\ a_1 \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \\ a_2 \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \\ a_3 \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} a_0b_0 \\ a_0b_1 \\ a_0b_2 \\ a_1b_0 \\ a_1b_1 \\ a_1b_2 \\ a_2b_0 \\ a_2b_1 \\ a_2b_2 \\ a_3b_0 \\ a_3b_1 \\ a_3b_2 \end{bmatrix}$$

In general $\mathbf{C}^m \times \mathbf{C}^n$ is much smaller than $\mathbf{C}^m \otimes \mathbf{C}^n$.

Separable versus Entangled

$$\begin{bmatrix}8\\12\\6\\12\\18\\9\end{bmatrix} \in \mathbf{C}^6 = \mathbf{C}^2 \otimes \mathbf{C}^3 = \begin{bmatrix}2\\3\end{bmatrix} \otimes \begin{bmatrix}4\\6\\3\end{bmatrix}$$

separable

$$\begin{bmatrix}8\\0\\0\\0\\0\\18\end{bmatrix} \in \mathbf{C}^6 = \mathbf{C}^2 \otimes \mathbf{C}^3 = \begin{bmatrix}x\\y\end{bmatrix} \otimes \begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}xa\\xb\\xc\\ya\\yb\\yc\end{bmatrix}$$

no solution!

$$\begin{bmatrix}8\\0\\0\\0\\0\\18\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}8\\0\\0\end{bmatrix} + \begin{bmatrix}0\\6\end{bmatrix} \otimes \begin{bmatrix}0\\0\\3\end{bmatrix}$$

entangled: sum of tensors

Tensor product of two matrices

$$A \otimes B = \begin{bmatrix}a_{0,0} & a_{0,1}\\ a_{1,0} & a_{1,1}\end{bmatrix} \otimes \begin{bmatrix}b_{0,0} & b_{0,1} & b_{0,2}\\ b_{1,0} & b_{1,1} & b_{1,2}\\ b_{2,0} & b_{2,1} & b_{2,2}\end{bmatrix}$$

$$= \begin{bmatrix} a_{0,0}\cdot\begin{bmatrix}b_{0,0} & b_{0,1} & b_{0,2}\\ b_{1,0} & b_{1,1} & b_{1,2}\\ b_{2,0} & b_{2,1} & b_{2,2}\end{bmatrix} & a_{0,1}\cdot\begin{bmatrix}b_{0,0} & b_{0,1} & b_{0,2}\\ b_{1,0} & b_{1,1} & b_{1,2}\\ b_{2,0} & b_{2,1} & b_{2,2}\end{bmatrix} \\ a_{1,0}\cdot\begin{bmatrix}b_{0,0} & b_{0,1} & b_{0,2}\\ b_{1,0} & b_{1,1} & b_{1,2}\\ b_{2,0} & b_{2,1} & b_{2,2}\end{bmatrix} & a_{1,1}\cdot\begin{bmatrix}b_{0,0} & b_{0,1} & b_{0,2}\\ b_{1,0} & b_{1,1} & b_{1,2}\\ b_{2,0} & b_{2,1} & b_{2,2}\end{bmatrix}\end{bmatrix}$$

$$= \begin{bmatrix}
a_{0,0}\times b_{0,0} & a_{0,0}\times b_{0,1} & a_{0,0}\times b_{0,2} & a_{0,1}\times b_{0,0} & a_{0,1}\times b_{0,1} & a_{0,1}\times b_{0,2}\\
a_{0,0}\times b_{1,0} & a_{0,0}\times b_{1,1} & a_{0,0}\times b_{1,2} & a_{0,1}\times b_{1,0} & a_{0,1}\times b_{1,1} & a_{0,1}\times b_{1,2}\\
a_{0,0}\times b_{2,0} & a_{0,0}\times b_{2,1} & a_{0,0}\times b_{2,2} & a_{0,1}\times b_{2,0} & a_{0,1}\times b_{2,1} & a_{0,1}\times b_{2,2}\\
a_{1,0}\times b_{0,0} & a_{1,0}\times b_{0,1} & a_{1,0}\times b_{0,2} & a_{1,1}\times b_{0,0} & a_{1,1}\times b_{0,1} & a_{1,1}\times b_{0,2}\\
a_{1,0}\times b_{1,0} & a_{1,0}\times b_{1,1} & a_{1,0}\times b_{1,2} & a_{1,1}\times b_{1,0} & a_{1,1}\times b_{1,1} & a_{1,1}\times b_{1,2}\\
a_{1,0}\times b_{2,0} & a_{1,0}\times b_{2,1} & a_{1,0}\times b_{2,2} & a_{1,1}\times b_{2,0} & a_{1,1}\times b_{2,1} & a_{1,1}\times b_{2,2}
\end{bmatrix}$$

The Leap from Classical to Quantum

Classical Deterministic Systems

- 6 vertices in a graph
- 27 marbles

0 • 6　　1 • 2　　2 • 1

3 • 5　　4 • 3　　5 • 10

- $X = [6,2,1,5,3,10]^T$

Classical Deterministic Systems

- 6 vertices in a graph
- 27 marbles

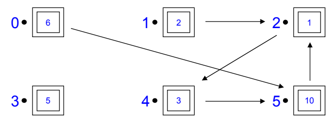
0 • 5　　1 • 5　　2 • 0

3 • 2　　4 • 0　　5 • 15

- $X = [5,5,0,2,0,15]^T$

Dynamics

- arrow from vertex *i* to vertex *j*: in one time click all marbles on vertex *i* will shift to vertex *j*



- Boolean adjacency matrix *M*

$$M = \begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&1&0&0&0&1\\0&0&0&1&0&0\\0&0&1&0&0&0\\1&0&0&0&1&0\end{bmatrix}$$

$M[i,j]=1$

↕

arrow from *j* to *i*
(see later)

$$MX = \begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&1&0&0&0&1\\0&0&0&1&0&0\\0&0&1&0&0&0\\1&0&0&0&1&0\end{bmatrix}\begin{bmatrix}6\\2\\1\\5\\3\\10\end{bmatrix} = \begin{bmatrix}0\\0\\12\\5\\1\\9\end{bmatrix} = Y$$

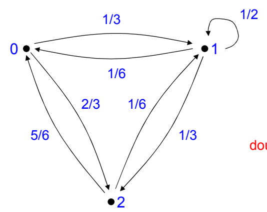
Dynamics (cont'd)

- In general:
  $M^k[i,j] = 1$ if and only if there is a path of length k from vertex j to vertex i.
- In Quantum Computing we start with an initial state (vector of numbers), the "input" of the system. Operations correspond with multiplying the vector with matrices. The "output" is the state of the system when all operations are carried out.
- Summing up:
  – The states of a system correspond to column vectors (state vectors).
  – The dynamics of a system correspond to matrices.
  – To progress from one state to another in one time step, one must multiply the state vector by a matrix.
  – Multiple step dynamics are obtained via matrix multiplications.

Probabilistic systems

- Quantum mechanics:
  – Inherent indeterminacy in knowledge of a state
  – States change with probabilistic laws
  – States transfer with a certain likelihood.
- Instead of many marbles, just look at one:
  – $X = [1/5, 3/10, 1/2]^T$ corresponds with
    - 1/5 chance that marble is on vertex 0
    - 3/10 chance that marble is on vertex 1
    - 1/2 chance that marble is on vertex 2
    - sum must be 1.

Modified dynamics



$$\text{Adjacency matrix } M = \begin{bmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

doubly stochastic: ∑rows =1 and ∑columns=1

$$MX = \begin{bmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{21}{36} \\ \frac{9}{36} \\ \frac{6}{36} \end{bmatrix} = Y$$

X is probability at time t

Y is probability at time $t+1$

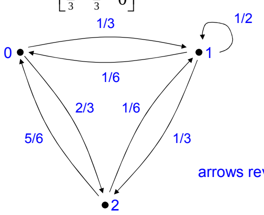
Symmetry

- Multiplication also on the left of a matrix with a row vector (=state vector):

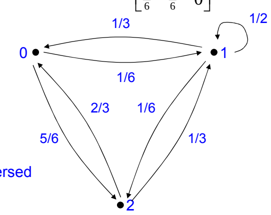
$$WM = \begin{bmatrix} \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{9} & \frac{5}{18} & \frac{5}{18} \end{bmatrix} = Z$$

$$\text{Note}: \sum \text{entries } Z = 1$$

$$M = \begin{bmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} \qquad \text{Transpose } M^T = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ \frac{5}{6} & \frac{1}{6} & 0 \end{bmatrix}$$



arrows reversed



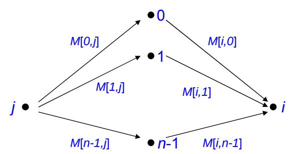
$$M^T W^T = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ \frac{5}{6} & \frac{1}{6} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{9} \\ \frac{5}{18} \\ \frac{5}{18} \end{bmatrix} = Z^T \qquad M^T W^T = (WM)^T = Z^T$$

multiply on the right of M takes states from time t to time $t+1$

multiply on the left of M takes states from time t to time $t-1$

time symmetry

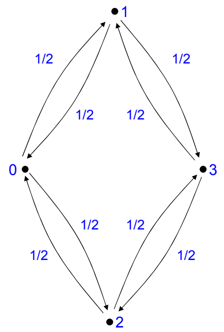
$$MM = M^2 = \begin{bmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} = \begin{bmatrix} \frac{11}{18} & \frac{13}{36} & \frac{1}{36} \\ \frac{5}{18} & \frac{13}{36} & \frac{13}{36} \\ \frac{1}{9} & \frac{5}{18} & \frac{11}{18} \end{bmatrix}$$



$M^2[i,j]$ = the probability of going from vertex j to vertex i in 2 time clicks.

In general for each positive integer k, we have $M^k[i,j]$ = the probability of going from vertex j to vertex i in k time clicks.

The stochastic billiard ball



$$A = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

start single marble in vertex 0: $[1,0,0,0]^T$

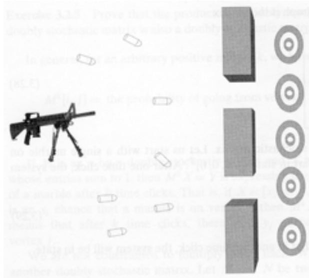
after one time click: $[0,1/2,1/2,0]^T$

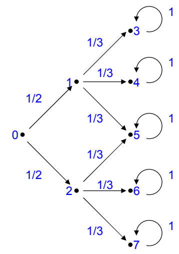
after another time click: $[1/2,0,0,1/2]^T$

marble acts like a billiard ball that bounces back and forth between vertices 1,2 and 0,3

quantum version will follow

Probabilistic double-slit experiment (I)





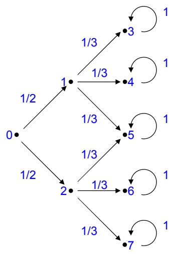
bullet always through one of the two slits

50% chance through top slit, 50% chance through bottom slit

three targets after each slit that can be hit with equal probability

one time click to a slit, one to a target

Probabilistic double-slit experiment (II)



$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Probabilistic double-slit experiment (III)

$$B * B = B^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

B^2 probabilities of bullet's position after two time ticks

Sure that we start with bullet in position 0: $X = [1,0,0,0,0,0,0,0]^T$

After two time clicks: $B^2X = [0,0,0,1/6,1/6,1/3,1/6,1/6]^T$

$B^2[5,0] = 1/6 + 1/6 = 1/3$, what we expect.

In QM strange things……

Summarizing

- The vectors that represent states of a probabilistic physical system express a type of indeterminacy about the exact physical state of the system.
- The matrices that represent the dynamics express a type of indeterminacy about the way the physical system will change over time. Their entries enable us to compute the likelihood of transitioning from one state to the next.
- The way in which the indeterminacy progresses is simulated by matrix multiplication, just as in the deterministic scenario.

Quantum Systems

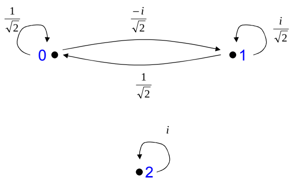
- QM: weight is not a real number *p* between 0 and 1, rather a complex number *c* such that $|c|^2$ is a real number between 0 and 1.
- Real number probabilities can only increase when added; complex numbers can cancel each other and lower their probability. This is called interference.

States and Graphs

- States: not sum of entries, but the sum of the modulus squared should be 1.

$$X = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{2i}{\sqrt{15}} & \sqrt{\frac{2}{5}} \end{bmatrix}^T$$

- Graphs: not with real number weights, but with complex number weights. Adjacency matrix not double stochastic, but unitary.



$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & i \end{bmatrix}$$

$$|U[i,j]|^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$UX = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & i \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{2i}{\sqrt{15}} \\ \sqrt{\frac{2}{5}} \end{bmatrix} = \begin{bmatrix} \frac{5+2i}{\sqrt{30}} \\ \frac{-2-\sqrt{5}i}{\sqrt{30}} \\ \sqrt{\frac{2}{5}}i \end{bmatrix} = Y$$

note: sum of the modulus squares of Y is 1

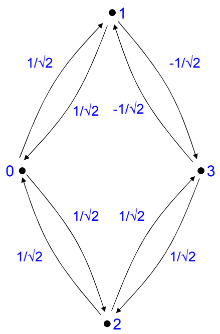
If U is the matrix that takes a state from time t to time $t+1$,

then $U^\dagger$ is the matrix that takes a state from time t to time $t-1$.

$V \rightarrow UV \rightarrow U^\dagger UV = I_3 V = V$

"undo" the operation

The quantum billiard ball



$$A = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

start single marble in vertex 0: $[1,0,0,0]^T$

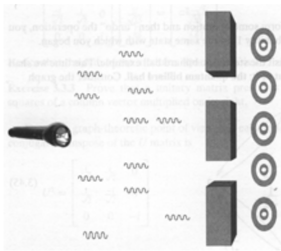
after one time click: $[0,\sqrt{1/2},\sqrt{1/2},0]^T$: 50-50%

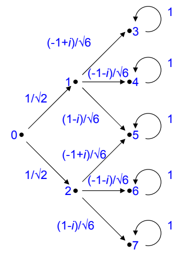
after another time click: $[1,0,0,0]^T$

other paths cancel each other out (interference)

$A^*A = A^2 = I_4$

Double-slit experiment (I)





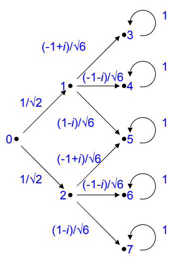
photons follow laws of quantum physics

each slit has a 50% chance of the photon's passing through it

three measuring devices after each slit

one time click from laser to wall, one from wall to measuring devices

Double-slit experiment (II)



$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

not unitary: many other paths

Double-slit experiment (III)

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|P[i,j]|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

exactly the same as with bullets: nothing strange happens after one time click.

Double-slit experiment (IV)

$$P^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1+i}{\sqrt{12}} & \frac{-1+i}{\sqrt{6}} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1-i}{\sqrt{6}} & \frac{-1+i}{\sqrt{6}} & 0 & 0 & 1 & 0 & 0 \\ \frac{-1-i}{\sqrt{12}} & 0 & \frac{-1-i}{\sqrt{6}} & 0 & 0 & 0 & 1 & 0 \\ \frac{1-i}{\sqrt{12}} & 0 & \frac{1-i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left|P^2[i,j]\right|^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Almost exactly as B^2, but one glaring difference: $B^2[5,0] = 1/3$, but $|P^2[5,0]|^2 = 0$

Explanation

- Interference: waves?
- However, experiment can be done with a *single* photon: interference!
- Superposition: *all* positions simultaneously!
- Measurement: no longer superposition, but collapse to a single classical state.

Review

- States in a quantum system are represented by column vectors of complex numbers whose sum of moduli squared is 1.
- The dynamics of a quantum system is represented by unitary matrices and is therefore reversible. The "undoing" is obtained via the algebraic inverse, i.e., the adjoint of the unitary matrix representing forward evolution.
- The probabilities of quantum mechanics are always given as the modulus square of complex numbers.
- Quantum states can be superposed, i.e., a physical system can be in more than one basic state simultaneously.

Errata

All errata:
http://www.cambridge.org/resources/0521879965/7337_Errata.pdf

This link can be found on the QC-webpage.

Reading

- This lecture: Ch 2.5-2.7 & Ch 3.1-3.3, p 60-97.
- Next lecture: Ch 3.4 & (start) Ch 4.