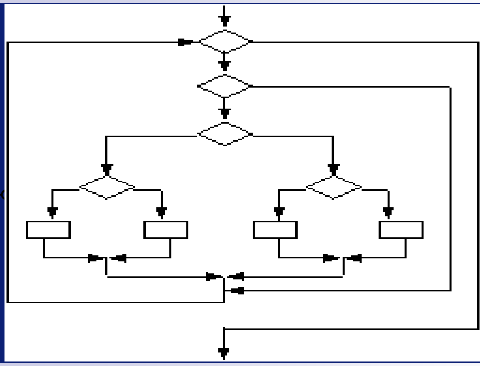
Spring 2007



Program correctness

Transition systems

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Formal Verification

Verification techniques comprise

- a modelling framework M, Γ

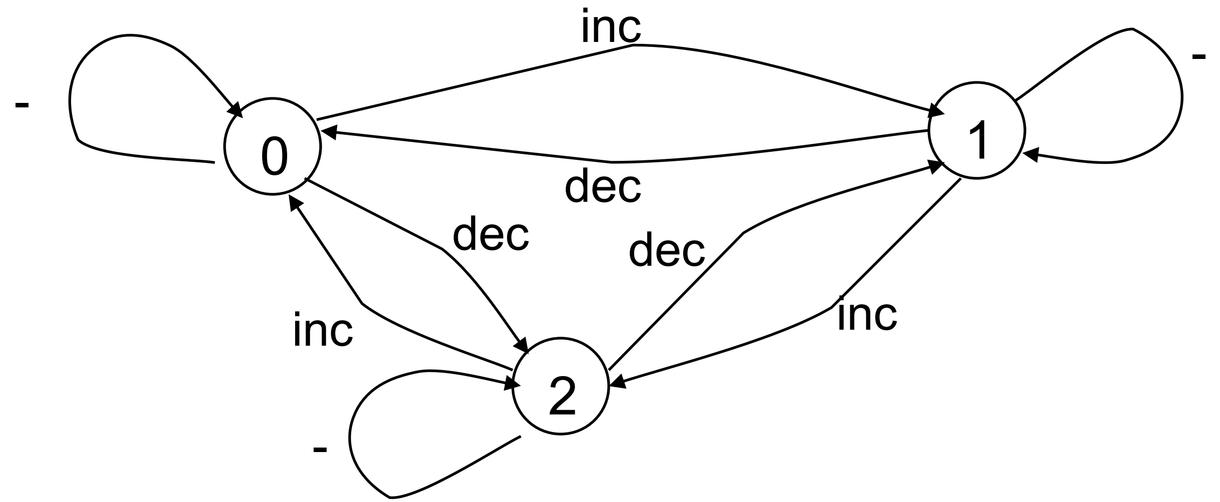
  to describe a system
- a specification language ϕ

  to describe the properties to be verified
- a verification method $M \models \phi, \Gamma \vdash \phi$

  to establish whether a model satisfies a property

Transition Systems

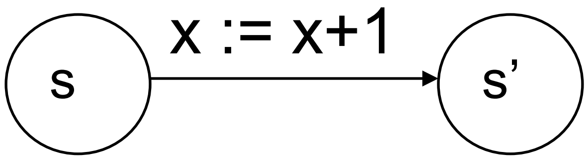
- A very general modelling framework
- Intuitively: a system evolves from one state to another under the action of a transition



A modulo 3 counter

Example: an assignment

States: s:Var -> Val

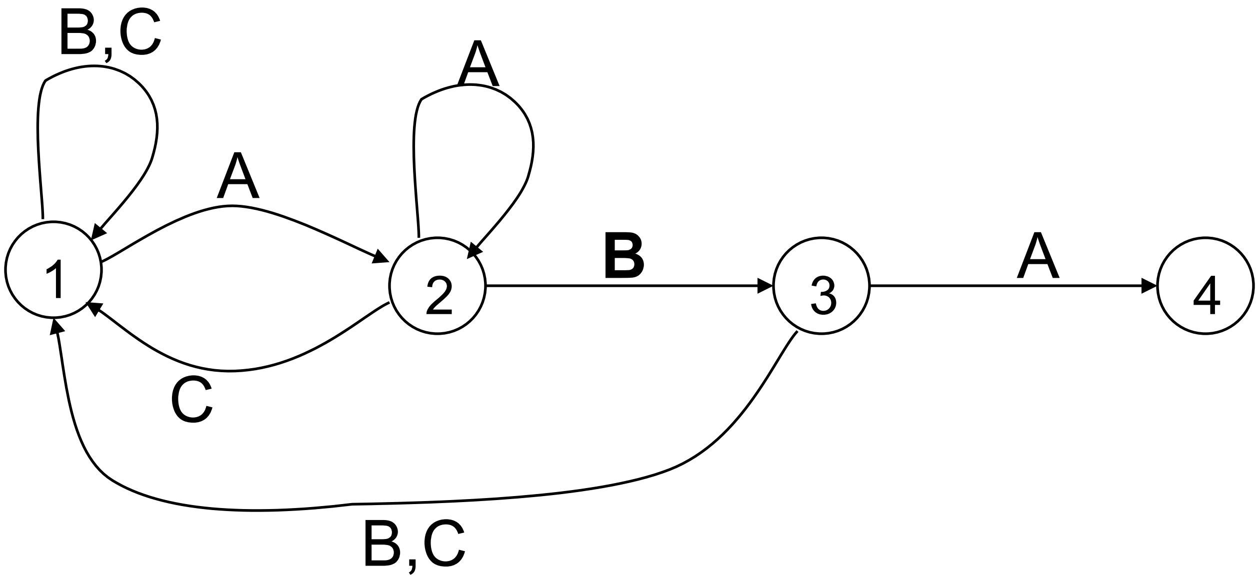


where s'=s[s(x)+1/x] and

$$f[v/x](y) = \begin{cases} f(y) & \text{if } x \neq y \\ v & \text{if } x = y \end{cases}$$

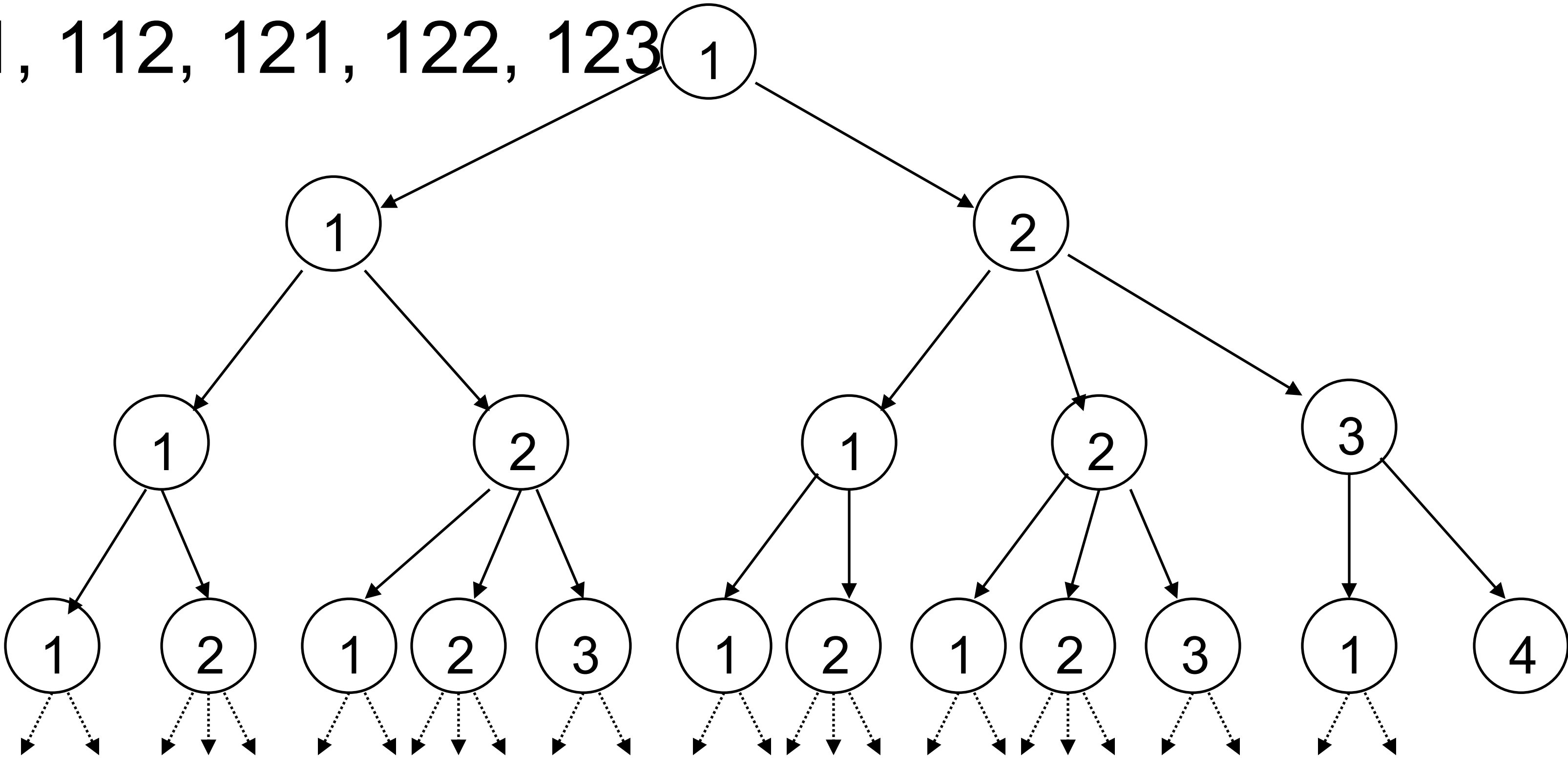
Example: a digicode

- 3 keys: A, B, C
- The door open when ABA is keyed



Digicode's executions

- 1
- 11, 12
- 111, 112, 121, 122, 123
- ...



A few definitions

- Transition system: <S,L,→>
  - S set of states
  - L set of transition labels
  - $\rightarrow \subseteq S \times L \times S$ transition relation
- Path: a sequence π of infinite transitions which follow each other

  For example

  $$3 \xrightarrow{B} 1 \xrightarrow{A} 2 \xrightarrow{A} 2 \ldots$$

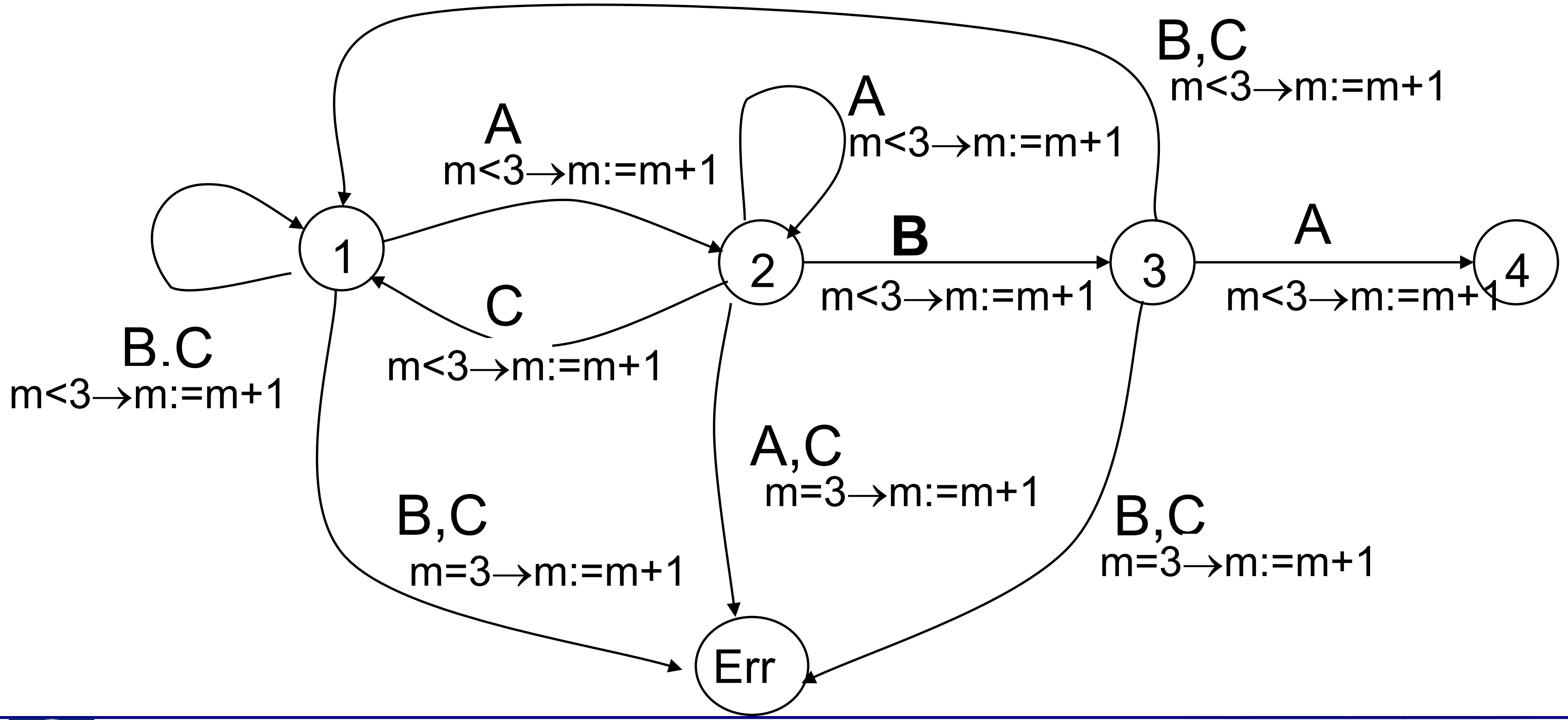
  is a path of the digicode

Adding data

- Real-life systems consist of control and data. We can model them by
  - control = states+transitions
  - data = state variables
- A transition system interact with state variables in two ways
  - guards a transition cannot occur if the condition does not holds
  - assignment a transition can modify the value of some state variables

Back to the digicode

- We do not tolerate more than 3 mistakes (recorded by the variable m)



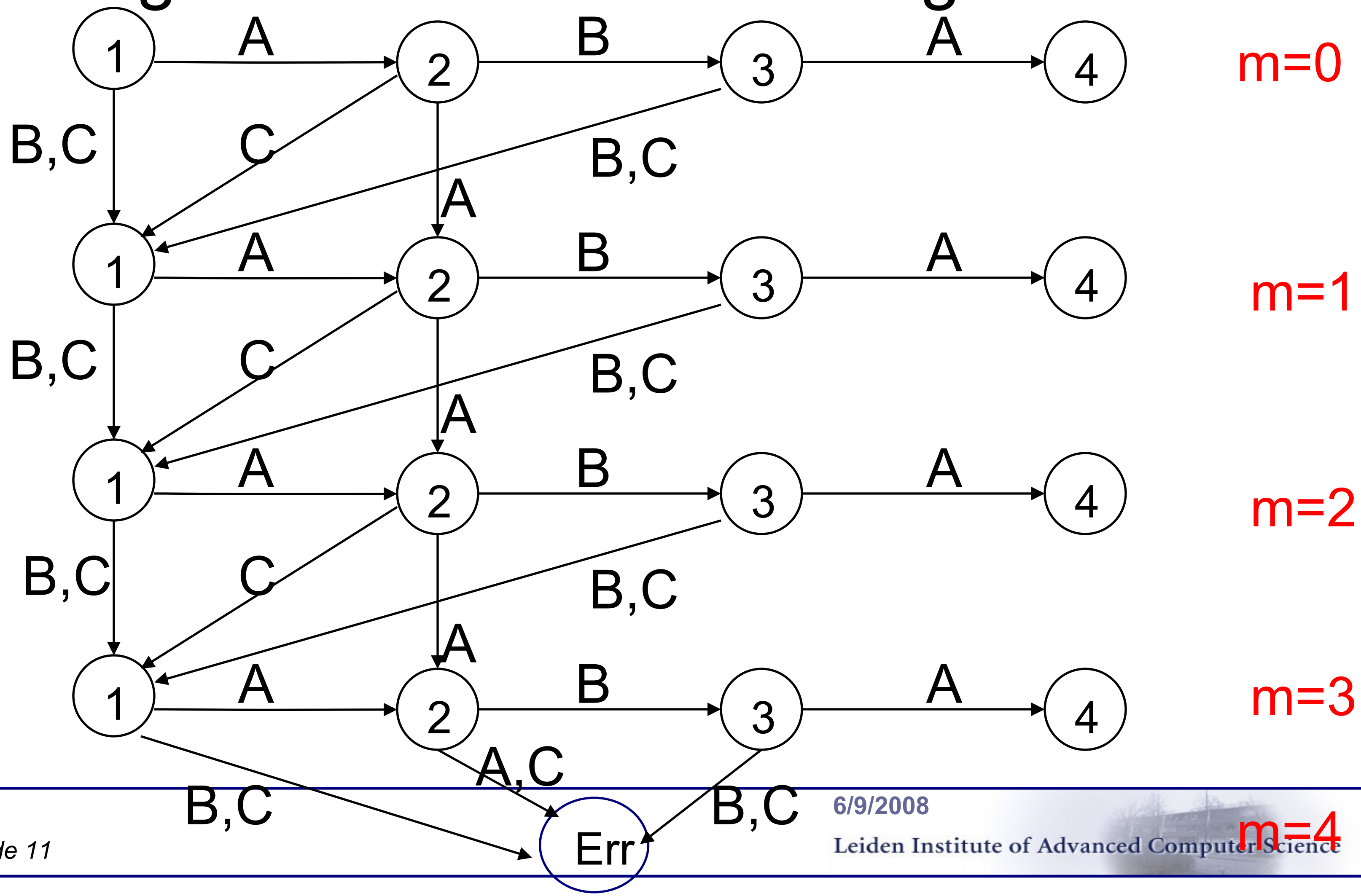
Unfolding

From a theoretical point of view, transition systems with state variables are not strictly necessary, as we can unfold them into ordinary transition systems.

- The new states correspond to the old ones + a component for each variable giving its value
- no more guards and assignment on the new transitions

Unfolding: example

- The digicode with error counting



Composing systems

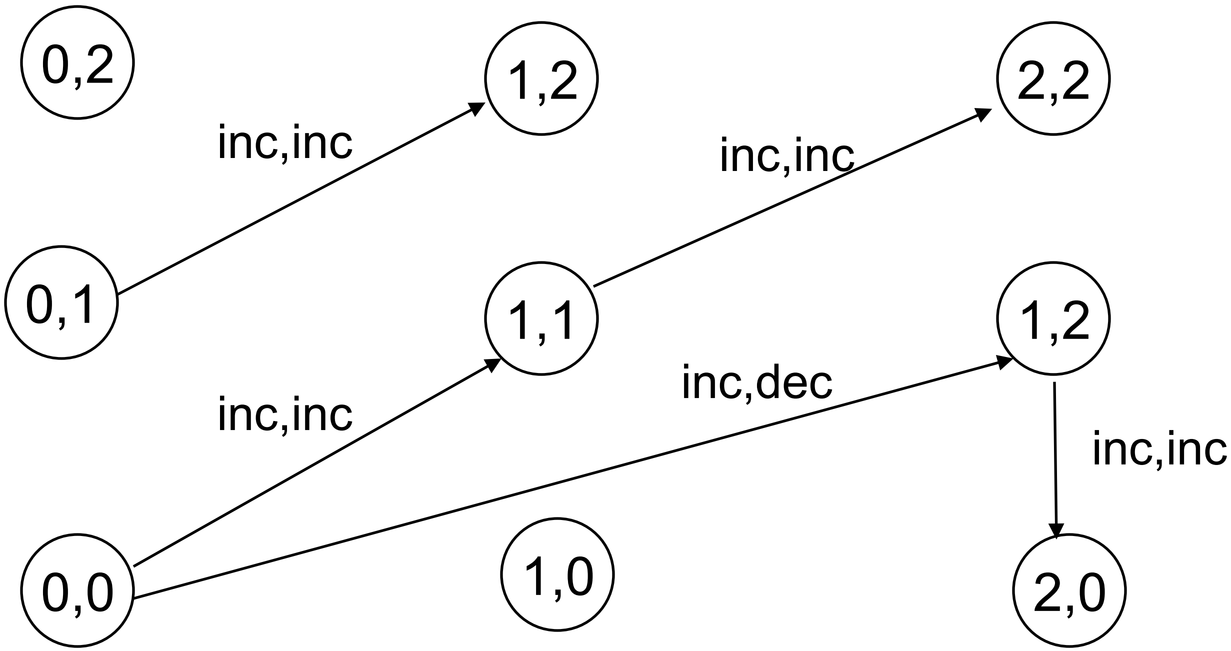
- Systems often consists of cooperating subsystems. Next we describe how to obtain a global transition system form its subsystem by having them cooperate
- There are many ways to cooperate:
  - product (no interaction)
  - synchronous product
    - by message passing
    - by asynchronous channels
    - by shared variables

Product

- Subsystems do not interact with each other
- The resulting transition system $<S,L,\rightarrow>$ is the cartesian product of the transition systems $<S1,L1,\rightarrow>$,…, $<Sn,Ln,\rightarrow>$ representing the subsystems
  - $S = S1 \times \ldots \times Sn$
  - $L = L1 \times \ldots \times Ln$
  - $<s_1,\ldots,s_n> \xrightarrow{<e_1,\ldots,e_n>} <t_1,\ldots t_n>$ if for all i, $s_i \xrightarrow{e_i} t_i$

Example

- Few transitions of the product of two modulo 3 counters



Synchronized Product

- Subsystems interact by doing some step together (synchronization).
- To synchronize subsystems we restrict the transitions allowed in their cartesian product.
- A synchronization set

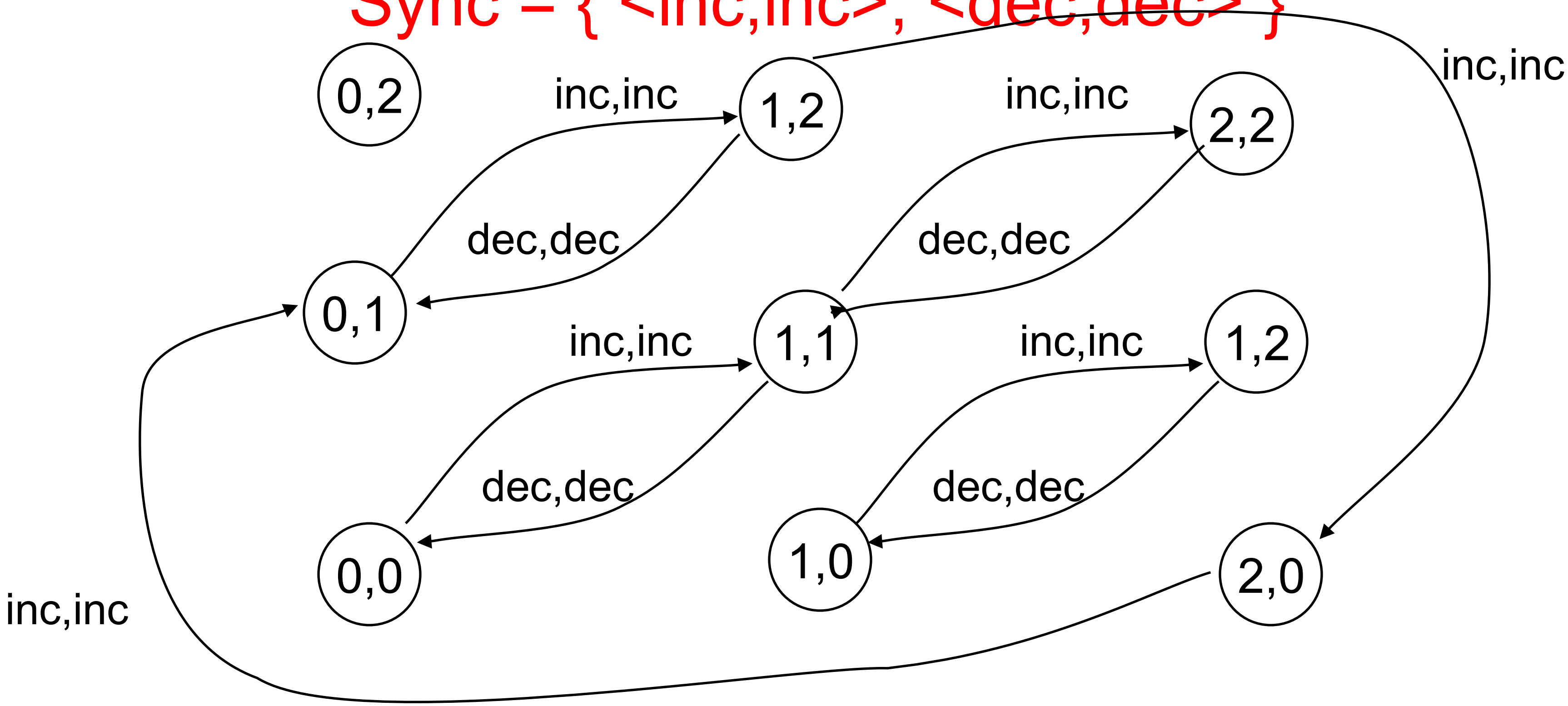
$$\text{Sync} \subseteq L1 \times \ldots \times Ln$$

define the labels of those transitions corresponding to a synchronization. Transitions with other labels are forbidden.

Example

- Few transitions of two counters counting at the same time

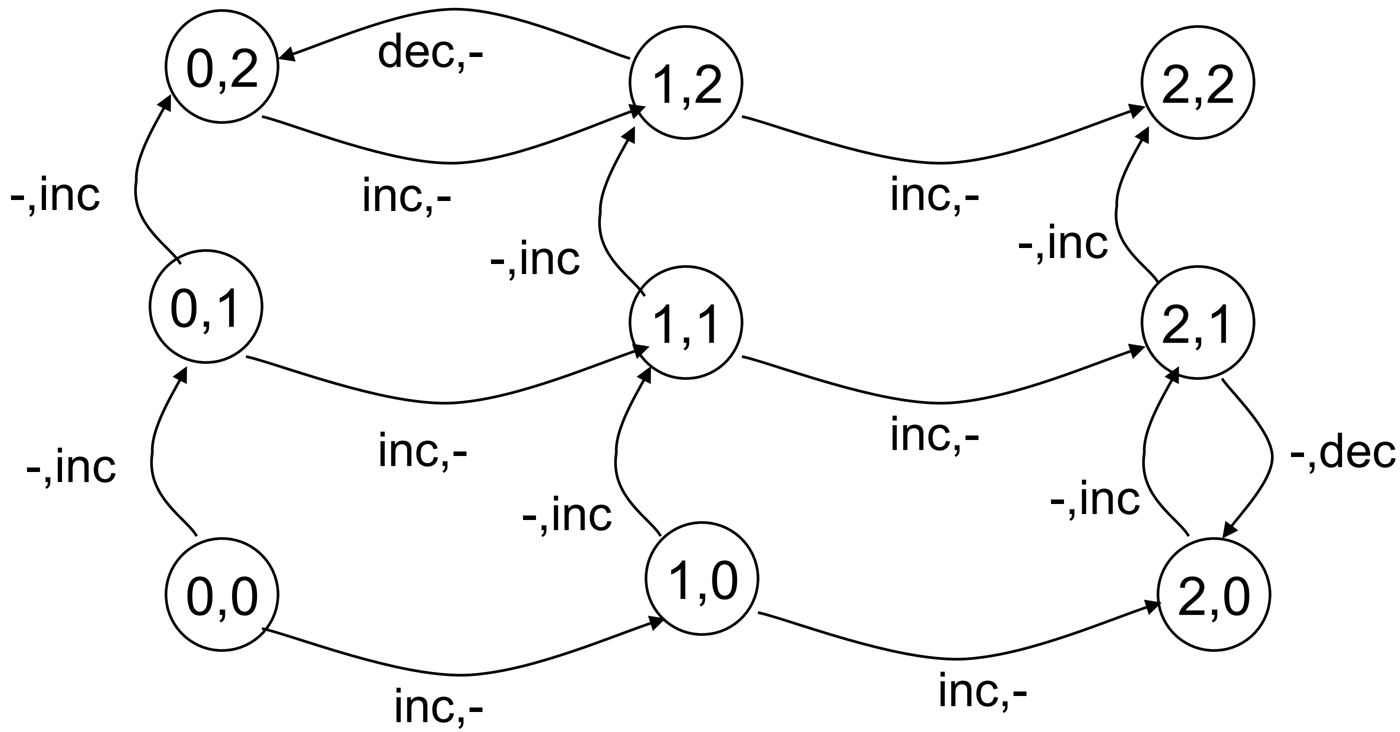
Sync = { <inc,inc>, <dec,dec> }



Example

- Few transitions of two counters counting one at the time

Sync = { <inc,->, <dec,->, <-,inc>, <-,dec>}



Message Passing

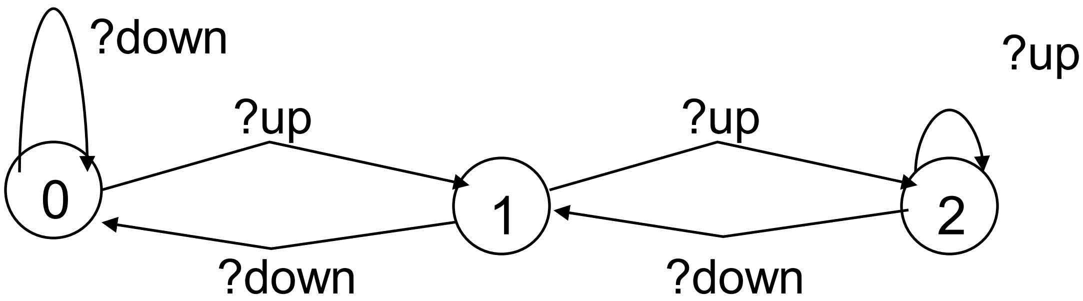
- A special case of synchronized product
- Two special sets of labels
  - !m emission of message m
  - ?m reception of message m
- In message passing, only transitions in which a given emission is executed simultaneously with the corresponding reception will be permitted

Example: An elevator

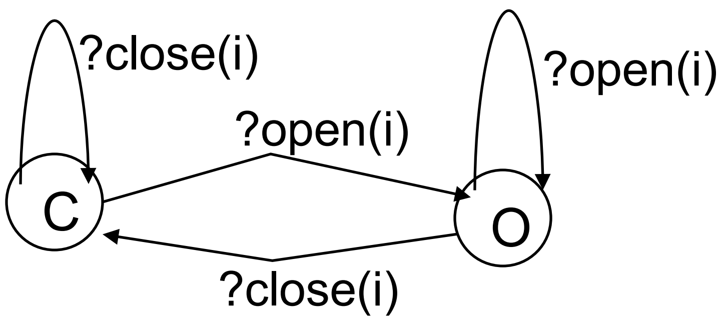
- An elevator in a three floors building consists of
  - a cabin which goes up and down
  - three doors which open an close
  - a controller which commands the three doors and the cabin
- Elevator requests from people at one of the three floors are not modeled, as they are the environment outside the system

Example: An elevator

- The cabin

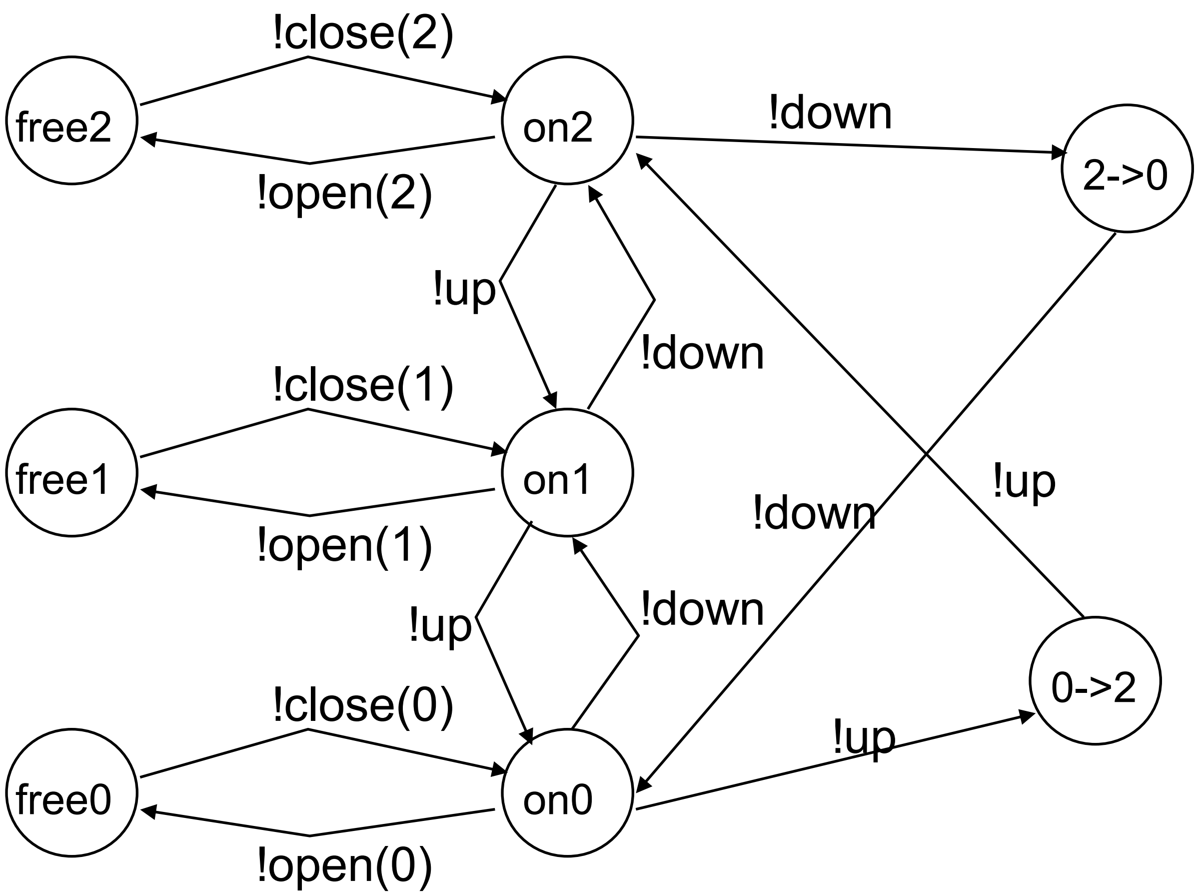


- The i-th door



Example: An elevator

- The controller



Example: An elevator

- The synchronization
  - Sync =

{<?open(0),-,-,-,!open(0)>,<?close(0),-,-,-,!close(0)>,

<-,?open(1),-,-,!open(1)>,<-,?close(1),-,-,!close(1)>,

<-,-,?open(2),-,!open(2)>,<-,-,?close(2),-,!close(2)>,

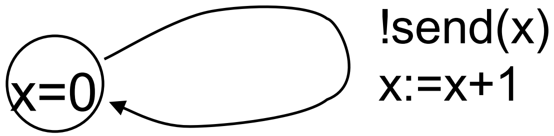
<-,-,-,?down,!down>,<-,-,-,?up,!up>}

Asynchronous Messages

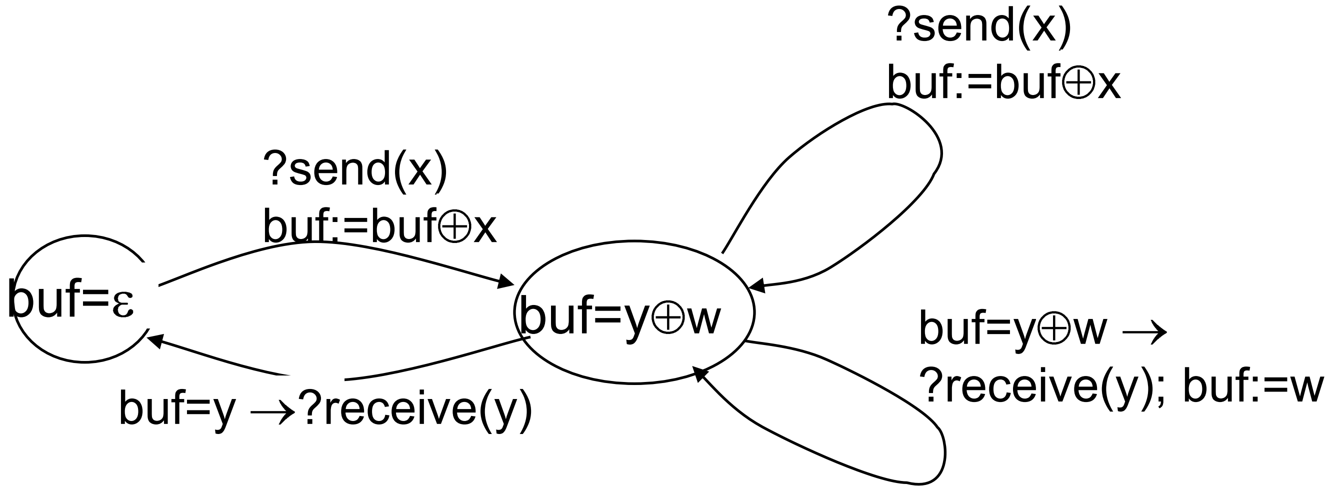
- Like message passing, but messages are not received instantly.
- Emitted messages but not yet received remain in a communication channel, usually a FIFO buffer
- A communication channel can be modeled by a transition system with a variable (for the buffer content)

Example:

- Producer



- Buffer



- Consumer